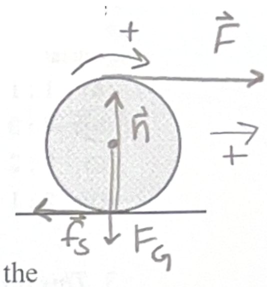
Solutions posted

Name: ___________
Date: ____________

Rotation Practice 2

1.



a. Draw a qualitatively-correct force diagram for the object on the picture to the right. (Is there static friction or kinetic friction? static To determine the direction of the friction force, consider the direction of the "tendency to slip" for the bottom of the object.)

b. Is there a non-zero sum of the forces in the horizontal or vertical direction? Explain the reasoning behind your answer.

Yes, there is a ΣF in the horizontal direction, because F has to be greater in magnitude than f_s in order to cause acceleration ($a_{cm} = \frac{\Sigma F}{m}$)

c. Is there a non-zero sum of the torques on the object? Explain the reasoning behind your answer.

Yes, since there is $\vec{a}_{cm}$ to the right, and it is rolling w/o slipping, $a_{cm} = r\alpha$, so there is angular acceleration. A $\Sigma\tau$ must be present to cause the angular acceleration.

d. Starting with fundamental physics principles, derive an expression for the acceleration of the object in terms of given variables and fundamental constants. (Set up a coordinate system with your + direction for translational and rotation motion on the force diagram or picture above.)

① $a_{cm} = \frac{F - f_s}{m}$

$m a_{cm} = F - f_s$

② $\alpha = \frac{\Sigma\tau}{I}$

$\left(\frac{a_{cm}}{r}\right) = \frac{+rF + rf_s}{kmr^2}$

$a_{cm} = \frac{F + f_s}{km}$

$km\, a_{cm} = F + f_s$

③ Add equations

$(m + km) a_{cm} = F - f_s + F + f_s$

$(m + km) a_{cm} = 2F$

$\boxed{a_{cm} = \frac{2F}{(m + km)}}$

2. The answer is B

$a = \frac{2F}{(m+km)}$

$a_s = \frac{2F}{(m + \frac{1}{2}m)}$

$a_s = \frac{2F}{(3/2)m}$

$a_s = \left(\frac{4}{3}\right)\frac{F}{m}$

$a_H = \frac{2F}{(m + 1m)}$

$a_H = \frac{F}{m}$

ratio:

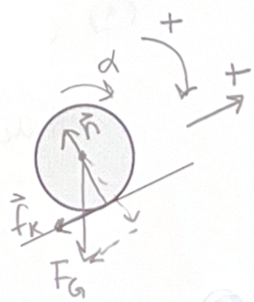
$\frac{a_s}{a_H} = \frac{(4/3)F/m}{F/m}$

$= \frac{4}{3}$

3.

a) The acceleration of the center of mass vector points down the ramp and the angular acceleration is clockwise. They do not have to be "in the same direction" because the object is rolling with slipping.

b) Draw a force diagram of the cylinder on the picture to the right. Think about which way the bottom of the object is sliding to determine the direction of the friction force. Choose a + direction for each and label them on the picture to the right.



$$a_{cm} \neq \alpha r$$

c) Derive an expression for the translational acceleration $\vec{a}_{CM}$

$$a_{cm} = \frac{\Sigma F}{m}$$

$$a_{cm} = \frac{-mg\sin\theta_R - \mu_K mg\cos\theta_R}{m}$$

$$\boxed{a_{cm} = -g\sin\theta_R - \mu_K g\cos\theta_R}$$

d) Derive an expression for the angular acceleration

$$\alpha = \frac{\Sigma \tau}{I}$$

$$\alpha = \frac{r_c f_K}{m_c r_c^2}$$

$$\alpha = \frac{\mu_K mg\cos\theta}{mr_c}$$

$$\boxed{\alpha = \frac{\mu_K g\cos\theta_R}{r_c}}$$

e) The answer is D

$$\frac{a_{cm}}{\alpha} = \frac{-g\sin\theta_R - \mu_K g\cos\theta_R}{\frac{(\mu_K g\cos\theta_R)}{r_c}}$$

$$\frac{a_{cm}}{\alpha} = \frac{-r_c(\cancel{g}\sin\theta_R + \mu_K \cancel{g}\cos\theta_R)}{\mu_K \cancel{g}\cos\theta_R}$$

$$\boxed{a_{cm} = -\alpha r_c \frac{(\sin\theta_R + \mu_K\cos\theta_R)}{\mu_K\cos\theta_R}}$$

Note: The negative sign is from the direction I set up my coordinate system.