Name: ____________
Date: ____________

Rotation Practice 1

1. Write these new physics facts into your booklet

Angular Momentum of a rigid body:

- The angular momentum of a rigid body with rotational inertia I and angular velocity $\vec{\omega}$ is $\vec{L} = I\vec{\omega}$
- An angular impulse occurs when a torque is exerted for some time interval: $\vec{K} = \int_{t_i}^{t_f} \vec{\tau} dt$
- The law of conservation of angular momentum is: $\vec{L}_i + \Sigma\vec{K} = \vec{L}_f$

Rolling:

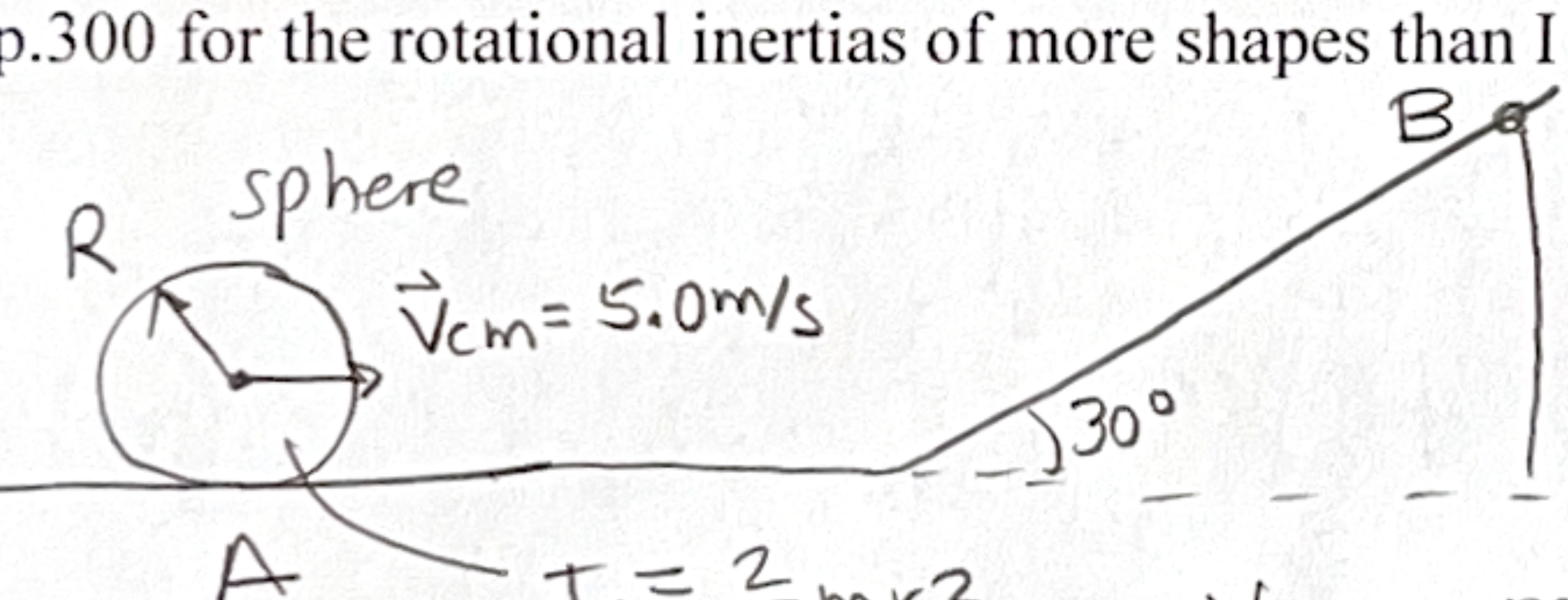
- An object that is rolling has both translational kinetic energy and rotational kinetic energy.
- If an object rolls without slipping, its rotational motion and the translational motion of its center of mass are related as follows:

  $x_{cm} = r\theta$
  $v_{cm} = r\omega$
  $a_{cm} = r\alpha$

Instructions for #2 and #3: For each problem, do Sketch and Translate, define your system, and state the physics principle you are applying. If your analysis has an initial time and a final time, be sure to label those.

2. p.334 #69

(See p.300 for the rotational inertias of more shapes than I included in your class notes.)



$I = \frac{2}{3}mr^2$

$V_{cm} = \omega r$ because it rolls w/o slipping, so $\omega = \frac{V_{cm}}{r}$

A: Rolling on floor
B: At max height

System:
floor
Sphere
Earth
ramp

Conservation of energy

$E_A + E_{transfers} = E_B$

$K_T + K_R = U_G$

$\frac{1}{2}mV_{cm}^2 + \frac{1}{2}I\omega^2 = mgh$

$\frac{1}{2}\cancel{m}V_{cm}^2 + \frac{1}{2}\left(\frac{2}{3}\cancel{m}r^2\right)\left(\frac{V_{cm}}{r}\right)^2 = \cancel{m}gh$

$\frac{1}{2}V_{cm}^2 + \frac{1}{3}V_{cm}^2 = gh$

$\frac{5}{6}V_{cm}^2 = gh$

$\boxed{h = \frac{5V_{cm}^2}{6g}}$

$h = \frac{(5)(5.0 m/s)^2}{6(10 N/kg)}$

$h = \boxed{2.1 m}$

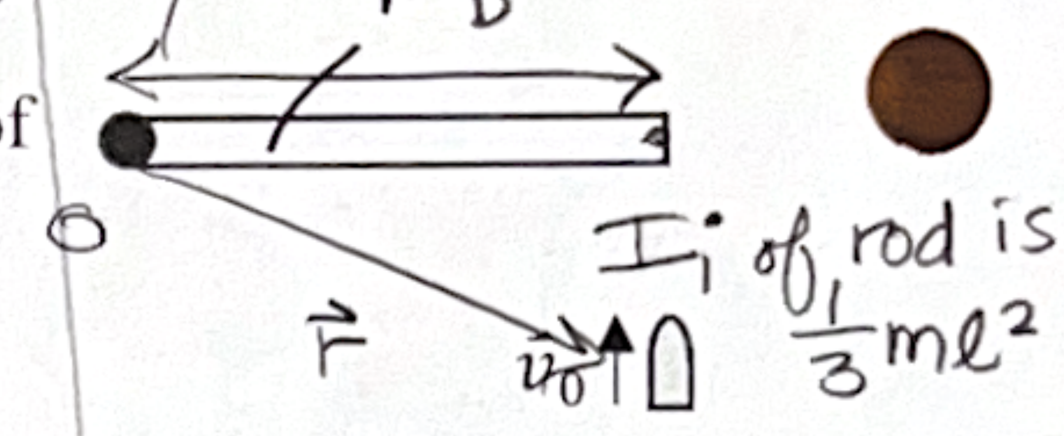
Ext forces would be whatever is supporting the floor + ramp. Those forces do no work, so $E_{transfers} = 0$.

Sketch + translate

Initial

$L = 1.0\,m$

$m_D = 10\,kg$



I_i of rod is $\frac{1}{3} m l^2$

$m_B = 10\,g$

$\vec{V}_0 = 400\,m/s$

Find ω just after impact.

3. p.334 #78

a. Do the problem as stated in the textbook. This picture shows a top-view of the door and bullet before the collision. (A door has the same rotational inertia as a rod pivoted about one end. Do you see why it does not matter how "high" the mass is distributed, only how far "out from the axle"?)

System: door, bullet

hinge, earth

external forces are from hinge and earth.

- Hinge causes no torque on door b/c $r=0$, so its $K=0$.
- Earth pulls ⊥ to the plane of motion + Δt is so short that its effect is negligible.

Conservation of Angular momentum

$$\vec{L}_i + \Sigma \vec{K} = \vec{L}_f$$

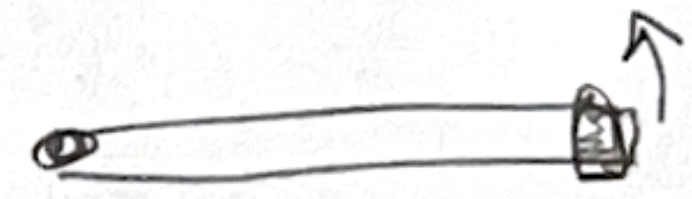
$$L_{Bullet} + \underbrace{L_{door}}_{0 \text{ b/c not moving}} + 0 = L_{(door+bullet)}$$

$$r_\perp m_B V_0 = I_f \omega$$

$$l\, m_B V_0 = \left(\frac{1}{3} m_D l^2 + m_B l^2\right) \omega$$

$$\boxed{\omega = \frac{m_B V_0}{l\left(\frac{1}{3} m_D + m_B\right)}}$$

Final



ω

Bullet

$I_f = I_{rod} + I_{bullet}$

$I_f = \frac{1}{3} m_D l^2 + m_B l^2$

$$\omega = \frac{(.010\,kg)(400\,m/s)}{(1.0\,m)\left(\frac{1}{3}(10\,kg) + .010\,kg\right)} = \boxed{1.2\,rad/s}$$

b. If the system is defined as the door and the bullet, calculate the change in kinetic energy of the system that occurred as a result of the collision. Why did the kinetic energy change?

System: door, bullet

hinge, earth

$$K_i = K_{Bullet} + \underbrace{K_{Door}}_{\text{This is 0}} = \frac{1}{2} m_B V_0^2 = \frac{1}{2}(.010\,kg)(400\,m/s)^2 = \boxed{800\,J}$$

$$K_f = I_{(Door+Bullet)}\,\omega = I_f\,\omega^2$$

$$K_f = \left(\frac{1}{3} m_D l^2 + m_B l^2\right)\omega^2$$

$$K_f = \left[\frac{1}{3}(10\,kg)(1\,m)^2 + (.010\,kg)(1\,m)^2\right]\left(1.2\,\frac{rad}{s}\right)^2$$

$$K_f = \boxed{4.8\,J}$$

$$\Delta K = K_f - K_i = 4.8\,J - 800\,J = \boxed{-795.2\,J}$$

The kinetic energy decreased because this is an inelastic collision and thermal energy increased.