Solutions posted

Name: ___________

Date: ___________

Practice: Center of Mass, Torque

1. Copy these new physics facts into your booklet:

- *Anything from the "Rotational Kinematics: Stationary Axle" notes that you want in your booklet and don't already have from the circular motion unit.*
- For a uniform rigid body, the center of mass is at the geometric center.
- For a system of point particles, the coordinates of the center of mass are $x_{CM} = \frac{m_1x_1 + m_2x_2 + \cdots}{m_1 + m_2 + \cdots}$ and $y_{CM} = \frac{m_1y_1 + m_2y_2 + \cdots}{m_1 + m_2 + \cdots}$
- The torque (turning ability) of a force about point O is $\vec{\tau} = \vec{r} \times \vec{F}$. Vector $\vec{r}$ is from point O to the point where the force is applied on the object
- The magnitude of torque is $\tau = rF\sin\phi$, where ϕ is the angle between the tails of $\vec{r}$ and $\vec{F}$. It can be calculated using perpendicular components: $\tau = r_{\perp}F$ or $\tau = rF_{\perp}$.

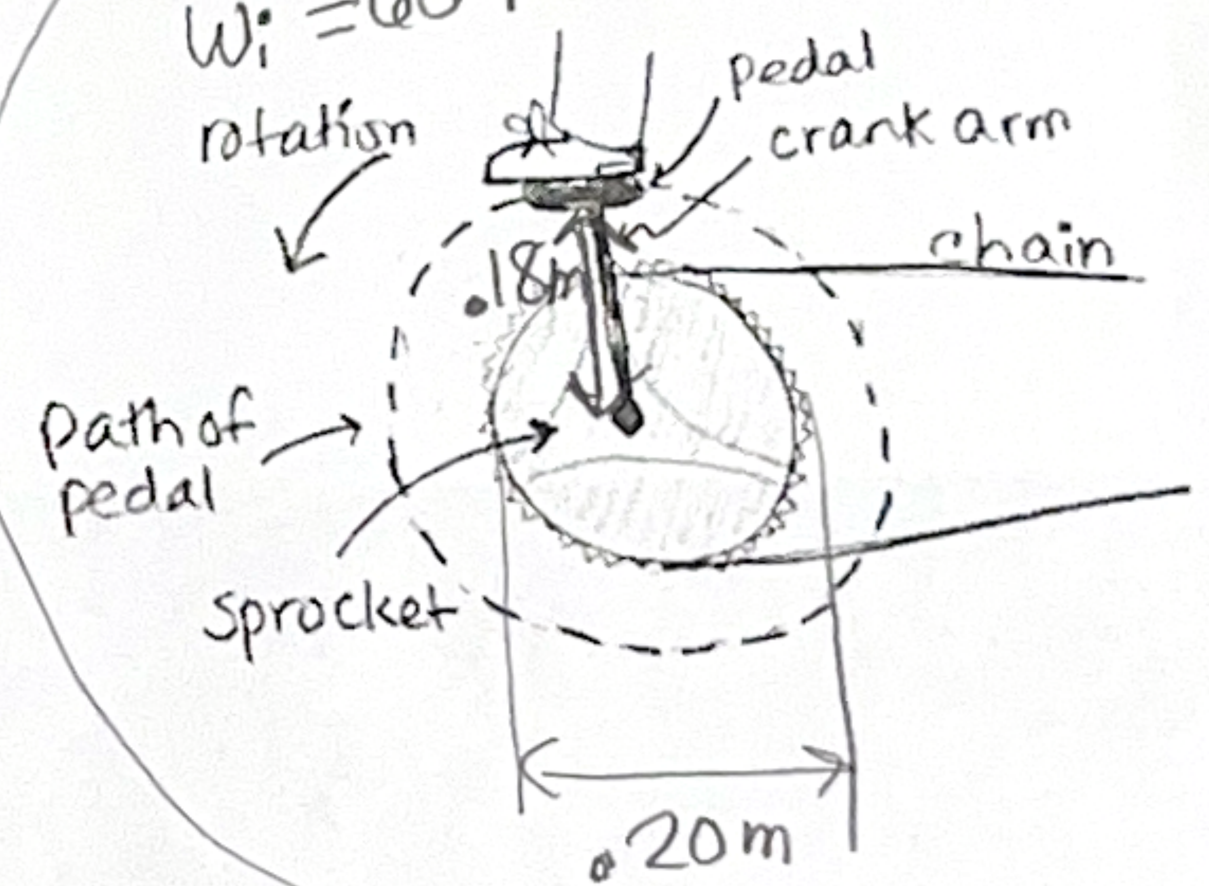
$\Delta t = 10s$

$\omega_f = 90\ rpm = 3\pi\ rad/s$

$\omega_i = 60\ rpm = 2\pi\ rad/s$

2. p. 330 #4 (To clarify what is happening, the sprocket rotates with the crank arm and pedal.)

- Sketch and translate (label the picture), then solve.
- Draw a tangential acceleration vector for the pedal on the picture.



a) Find a_t

For a rotating object with a fixed axle, $a_t = \alpha r$. So I need to find α.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{3\pi\ rad/s - 2\pi\ rad/s}{10s}$$

$$\alpha = .314\ rad/s^2$$

Now use $a_t = \alpha r$ to relate the angular acceleration of the crank arm + sprocket to the tangential acceleration of the pedal:

$$a_t = \alpha r$$

$$a_t = (.314\ rad/s^2)(.18m)$$

$$\boxed{a_t = .057\ m/s^2}$$

b) Find length of chain over the sprocket in this Δt:

The length of chain over the sprocket is equal to the arc length Δs that a point on the edge of the sprocket travels. So I need to find Δs.

Sprocket

$\Delta s = r\Delta\theta$. I need to know $\Delta\theta$ in order to find Δs. I can find $\Delta\theta$ from kinematics:

$$\Delta\theta = \omega_i\Delta t + \tfrac{1}{2}\alpha\Delta t^2$$

$$\Delta\theta = (2\pi\ \tfrac{rad}{s})(10s) + \tfrac{1}{2}(.314\ \tfrac{rad}{s^2})(10s)^2$$

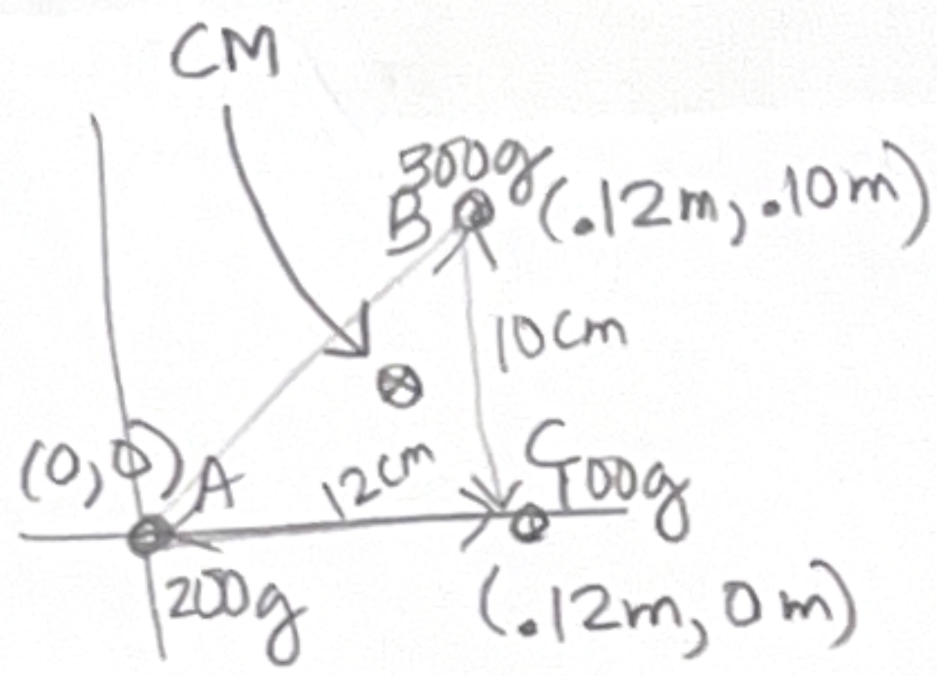
$$\Delta\theta = 78.5 rad$$

Now I can find Δs: $\Delta s = r\Delta\theta$

$$\Delta s = (.10m)(78.5\ rad)$$

$$\boxed{\Delta s = 7.85m}$$

3. p.330 #7. Sketch and translate, labeling the (x,y) coordinates of each object. Solve. Mark the position of the center of mass on your sketch.



$$X_{cm} = \frac{\cancel{m_A X_A}^{\,0} + m_B X_B + m_C X_C}{m_A + m_B + m_C}$$

$$y_{cm} = \frac{\cancel{m_A y_A}^{\,0} + m_B y_B + \cancel{m_C y_C}^{\,0}}{m_A + m_B + m_C}$$

$$X_{cm} = \frac{(300g)(.12m) + (100g)(.12m)}{200g + 300g + 100g}$$

$$y_{cm} = \frac{(300g)(.10m)}{200g + 300g + 100g}$$

$X_{cm} = .08m$

$y_{cm} = .5m$

$$\boxed{\text{Coordinates of CM are } (.08m, .5m)}$$