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Name: ____________

Date: ____________

Solutions posted

Practice: Rotational Motion- Work and Energy

1. Write these new facts into your booklet:

Rotational inertia:

- The rotational inertia of an object depends on its mass and how that mass is distributed.
- The general equation for rotational inertia is $I = kmr^2$, where k is constant that depends on the shape of the object.
- The rotational inertia of a point mass is $I = mr^2$
- The rotational inertia can be found by integrating over all elements of mass dm: $I = \int r^2 dm$

Work and energy for rotating objects:

- The gravitational potential energy depends on the position of the rigid body's center of mass.
- The rotational kinetic energy of a rotating rigid body is $K_{rot} = \frac{1}{2} I\omega^2$
- Rotational work is done when a torque acts through a displacement. The relationship is $W_{rot} = \int_{\theta_i}^{\theta_f} \vec{\tau} \cdot d\vec{\theta}$. For a constant torque, this reduces to $W_{rot} = \vec{\tau} \cdot \Delta\vec{\theta}$.
- The rotational work done by a torque on a rigid body is the area under the curve of a torque vs. angular position graph.

2. At a certain moment, a 143 g baseball that is flying through the air has a translational speed of 4.4 m/s (10 mph), while it is also spinning at an angular speed of 6π rad/s. Calculate the kinetic energy of the ball at this instant. (Model it as a solid sphere, and refer to the chart of rotational inertias from your class notes).

$r = 3.7cm$

Sphere: $I = \frac{2}{5}mr^2$

$I = \frac{2}{5}(.143kg)(.037m)^2$

$I = 7.8 \times 10^{-5}\ kg \cdot m^2$

$K = K_{trans} + K_{rot}$

$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

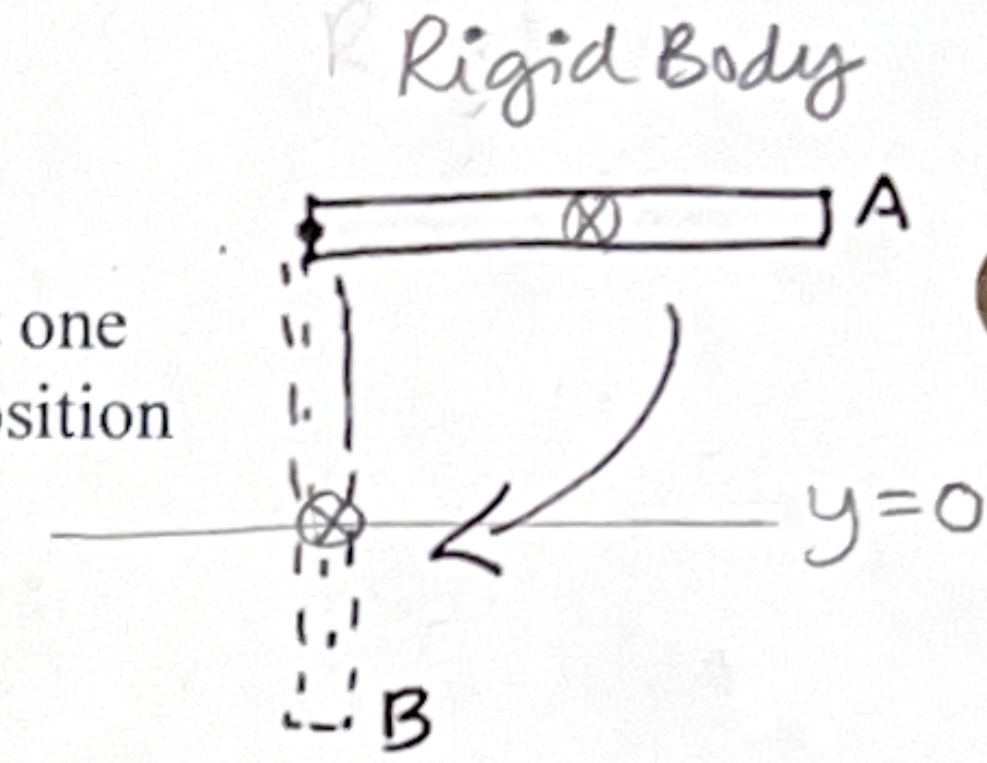
$K = \frac{1}{2}(.143kg)(4.4m/s)^2 + \frac{1}{2}(7.8\times10^{-5}kgm^2)(6\pi \frac{rad}{s})^2$

$K = 1.384\,J + .01386\,J$

$K = 1.398\,J$

$K = 1.4\,J$

3. A meterstick that has a mass of 100 g is pivoted about a fixed axis at one end. The meterstick is released from rest when it is in the horizontal position at A, and rotates down, moving at some angular speed when it passes through the vertical position at B.



a. What is the length of the meterstick? 1 m

b. Sketch & translate, state the object model, mark the object's center of mass, and label y = 0.

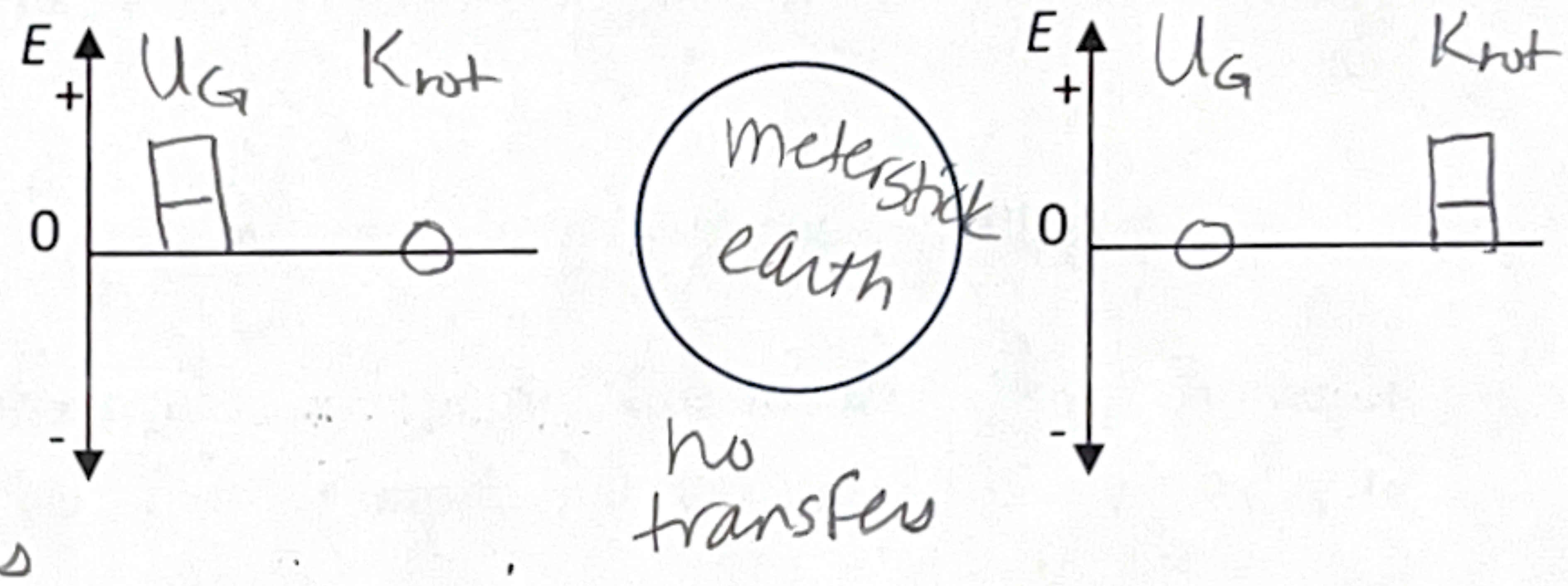
c. Calculate the rotational inertia of the meterstick.

$$I = \frac{1}{3} m\ell^2 = \frac{1}{3}(.1\,\text{kg})(1\,\text{m})^2 = \boxed{.0333\ \text{kg}\,\text{m}^2}$$

d. Define a system, do an external forces and work analysis (trans. and rot.), and complete the energy bar chart.

Initial time: A (Horiz)

Final time: B (vertical)



Ext forces & Work

- $\vec{F}_{axis}$
- The $\vec{F}_{axis}$ does no W_{trans} b/c it doesn't move stick through any $\Delta\vec{r}$. It does no W_{rot} b/c its torque is 0.

e. Calculate the angular speed of the meterstick when it passes through the vertical position at B.

$$E_i + \Sigma\,\text{transfers} = E_f$$

$$U_{GA} + 0 = K_{rot\,B}$$

$$mgh_A = \frac{1}{2} I \omega_B^2$$

$$mg\left(\frac{L}{2}\right) = \frac{1}{2} I \omega_B^2$$

$$\sqrt{\frac{mgL}{I}} = \omega_B$$

$$\omega_B = \sqrt{\frac{(.1\,\text{kg})(10\,\text{N/kg})(1\,\text{m})}{(.0333\,\text{kg}\,\text{m}^2)}}$$

$$\boxed{\omega_B = 5.5\ \text{rad/s}}$$

Units: $\sqrt{\dfrac{\text{kg}\left(\frac{\text{N}}{\text{kg}}\right)\text{m}}{\text{kg}\,\text{m}^2}} = \sqrt{\dfrac{\text{N}}{\text{kg}\cdot\text{m}}} = \sqrt{\dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}\cdot\dfrac{1}{\text{kg}\cdot\text{m}}} = \sqrt{\dfrac{1}{\text{s}^2}} = \dfrac{1}{\text{s}}$