(14)

Solutions posted

Name: ____________
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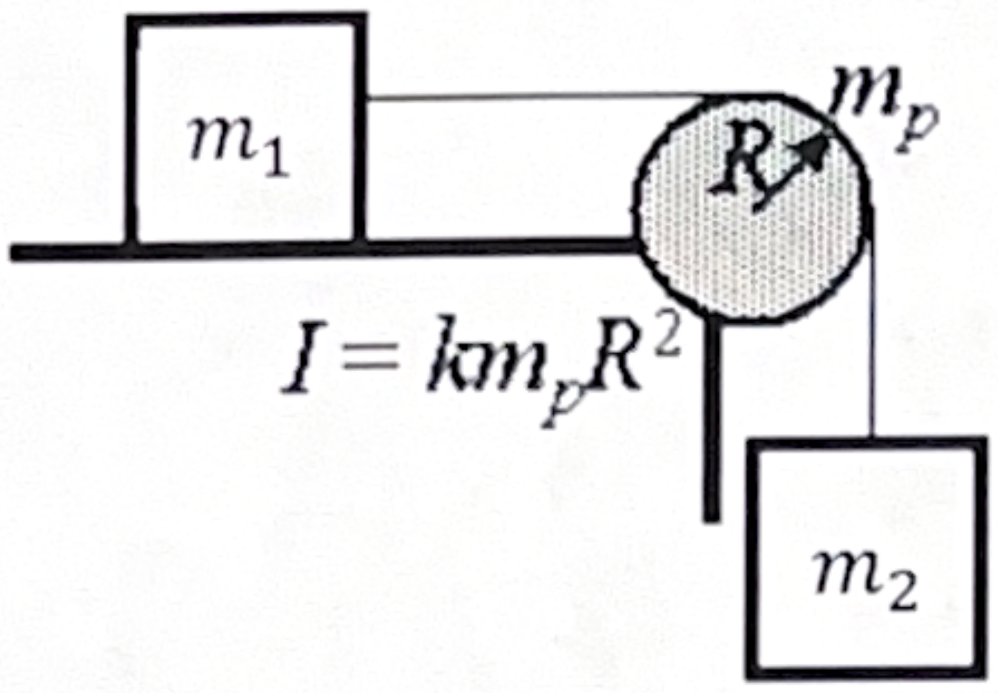
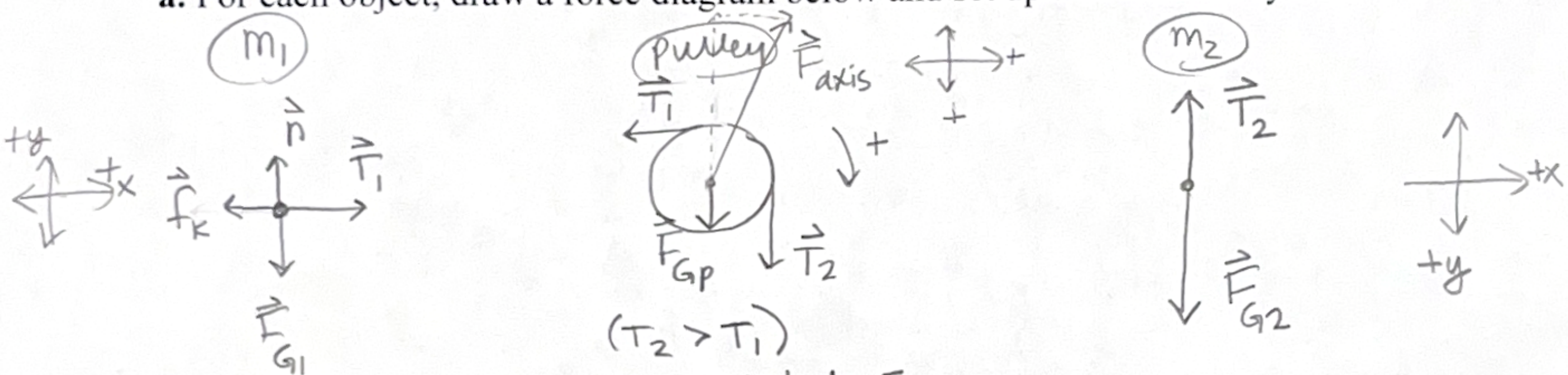
Practice: Newton's Second Law



Two blocks m_1 and m_2, are connected by a string that passes over a pulley of mass m_p, radius R, and a rotational inertia given by the equation $I = km_pR^2$. The string from which the masses hang is light and does not slip on the pulley. Block m_1 rests on a table where the coefficient of kinetic friction is μ_k. When block m_1 is released from rest, all the objects accelerate.

a. For each object, draw a force diagram below and set up a coordinate system.



$(T_2 > T_1)$
(See note below about $\vec{F}_{axis}$)

b. Complete a force organizer for each object:

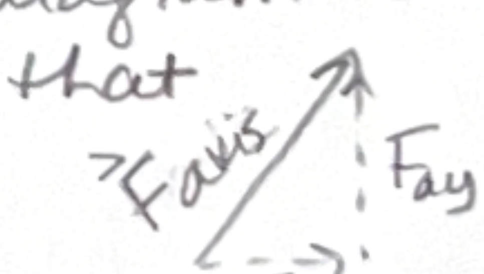
m_1:

$\vec{F}$	F_x	F_y	τ
$\vec{T}_1$	$T_{1x} = +T_1$	$T_{1y} = 0$	X
$\vec{n}$	$n_x = 0$	$n_y = +n$	X
$\vec{f}_k$	$f_{kx} = -\mu_k n$	$f_{ky} = 0$	X
$\vec{F}_{G1}$	$F_{Gx} = 0$	$F_{Gy} = -m_1 g$	X

pulley:

$\vec{F}$	F_x	F_y	τ
$\vec{T}_1$	$T_{1x} = -T_1$	$T_{1y} = 0$	$\tau_{T_1} = +RT_2$
$\vec{T}_2$	$T_{2x} = 0$	$T_{2y} = +T_2$	$\tau_{T_2} = -RT_1$
$\vec{F}_{Gp}$	$F_{Gpx} = 0$	$F_{Gpy} = +m_p g$	$\tau_{F_G} = 0$
$\vec{F}_{axis}$	$F_{axis\,x} = F_{ax}$	$F_{axis\,y} = F_{ay}$	$\tau_{axle} = 0$

Note:
For F_{axis}, since pulley's CM is not accelerating, $\Sigma\vec{F} = 0$. Draw vector addition diagram to see that



F_{axis} is at an angle.

m_2:

$\vec{F}$	F_x	F_y	τ
$\vec{T}_2$	X	$T_{2y} = -T_2$	X
$\vec{F}_{G2}$	X	$F_{Gy} = +m_2 g$	X

c. Derive an expression for the acceleration of the blocks below or on separate paper. Show your work.

For m_1

$a_{1x} = \frac{\Sigma F_{on 1}}{m_1}$

$a_1 = \frac{T_1 - \mu_k n}{m_1}$

But $n = m_1 g$, so

$a_1 = \frac{T_1 - \mu_k m_1 g}{m_1}$

For pulley

$\alpha = \frac{\Sigma \tau}{I}$

$\alpha = \frac{+RT_2 - RT_1}{k m_p R^2}$

$\alpha = \frac{R(T_2 - T_1)}{k m_p R^2}$

$\alpha = \frac{(T_2 - T_1)}{k m_p R}$

Express this as tangential accel, using
$a_t = \alpha r$
$\frac{a_t}{r} = \alpha$

$\frac{a_t}{R} = \frac{(T_2 - T_1)}{k m_p R}$

$a_t = \frac{(T_2 - T_1)}{k m_p}$

For m_2

$a_{2y} = \frac{\Sigma F_{on 2}}{m_2}$

$a_{2y} = \frac{m_2 g - T_2}{m_2}$

Now, $a_1 = a_t = a_{2y} = a$, so I can replace those variables all with a, and add the equations:

$$
\begin{aligned}
(m_1)a &= \cancel{T_1} - \mu_k m_1 g \\
+\ (k m_p)a &= \cancel{T_2} - \cancel{T_1} \\
+\ m_2 a &= m_2 g - \cancel{T_2} \\
\hline
a(m_1 + m_2 + k m_p) &= m_2 g - \mu_k m_1 g
\end{aligned}
$$

$$\boxed{a = \frac{m_2 g - \mu_k m_1 g}{(m_1 + m_2 + k m_p)}}$$

d. If the pulley was ideal, where its mass was negligible, the acceleration of the blocks comes out to be $a = \frac{m_2 g - \mu_k m_1 g}{(m_1 + m_2)}$. How does the equation for the acceleration of the blocks when the pulley *does* have mass (which you derived above) compare to this?

The only difference is in the denominator, where $k m_p$, a fraction of the pulley's mass, has been added to $m_1 + m_2$. As a result, the blocks have less acceleration than if the pulley was massless.