Solutions posted

Name: ________

Practice: Forces and Torques

Date: ________

1. Copy these new physics facts into your booklet:

- The sum of the forces on a rigid body is always in the same direction as the translational acceleration of its center of mass. (CM)
    - If $\Sigma\vec{F} = 0$, the CM of the rigid body remains at rest or translates with constant velocity.
    - If $\Sigma\vec{F} \neq 0$, the CM of the rigid body translates with constant acceleration.
- The sum of the torques on a rigid body about point O is always in the same direction as the angular acceleration of the rigid body about point O.
    - If $\Sigma\tau = 0$, the rigid body remains at rest or rotating with constant angular velocity.
    - If $\Sigma\tau \neq 0$, the rigid body is rotating with angular acceleration.
- *Static equilibrium* means that the rigid body remains at rest. For an object in static equilibrium, $\Sigma\vec{F} = 0$ and $\Sigma\tau = 0$.

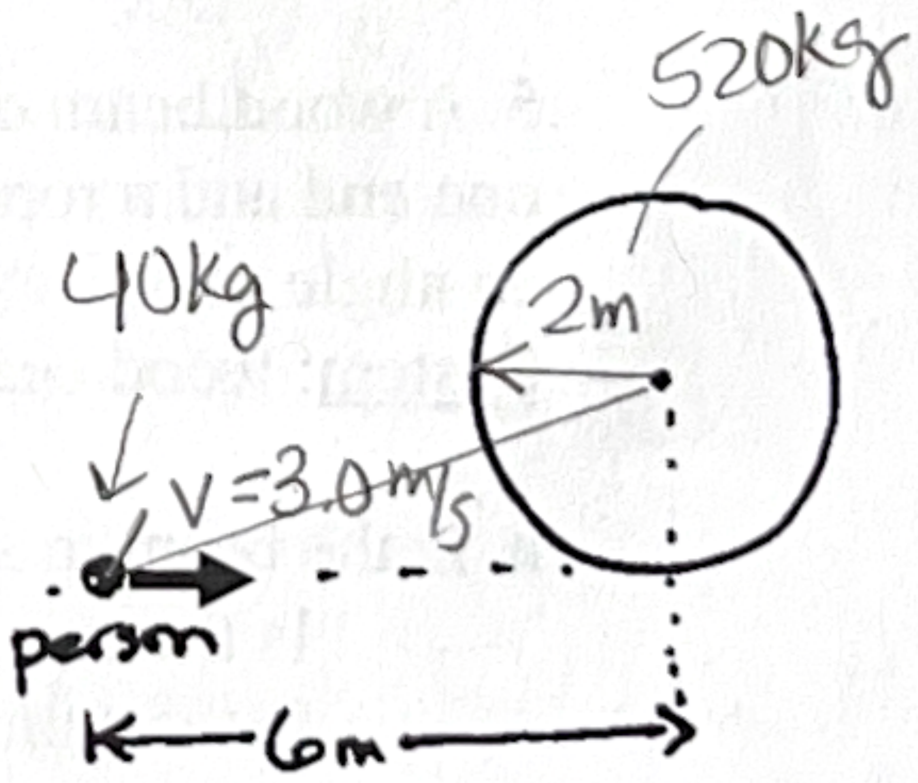
2. Turn to Analysis Problem 1, and do part (c). Parts (a) and (b) are not necessary for doing part (c), so once you read the problem statement, you are ready to answer (c).

$\vec{L} = r_{\perp} m v$

$L = (a)(mv_0)$

$\boxed{L = amv_0}$

3. A 40 kg child is running towards a playground merry-go-round with a speed of 3.0 m/s as shown in the aerial-view picture. She plans to jump on the edge when she reaches it. The merry-go-round has a diameter of 4 m and a mass 520 kg.



a. Sketch and translate by adding information to the picture.

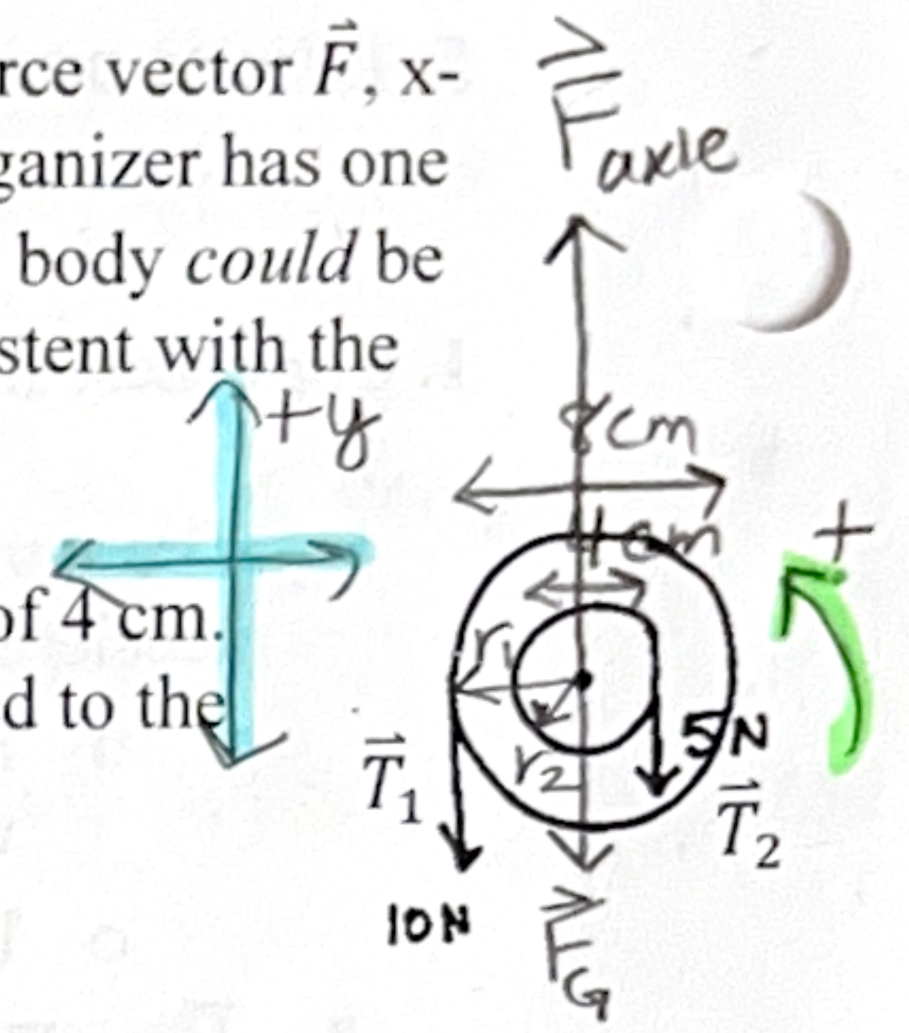
b. Calculate the angular momentum of the child about the center of the merry-go-round when she is at the position shown in the diagram. Include the direction (state CW / CCW or set up a coordinate system and give a unit vector)

$L = r_{\perp} m v = (2\text{m})(40\text{kg})(3.0\text{m/s}) = \boxed{240\ \text{kg m}^2/\text{s},\ \text{CCW}}$

c. Calculate the angular momentum of the child about the center of the merry-go-round when she just reaches the edge of the merry-go-round. Include the direction (state CW / CCW or set up a coordinate system and give a unit vector)

Since $\vec{L}$ only depends on $r_{\perp}$, m, and v, and all these are the same when she reaches the edge, $\boxed{\vec{L} = 240\ \text{kg m}^2/\text{s},\ \text{CCW}}$

Recall that for a **point particle**, the force organizer had these three columns: Force vector $\vec{F}$, x-scalar component F_x, and y-scalar component F_y. For a **rigid body**, the force organizer has one additional column for the scalar component of the torque vector, because a rigid body *could* be rotating! Each torque in the force organizer must have a + or – sign that is consistent with the rotational coordinate system you have set up.



4. A pulley of mass 300 g has an outer diameter of 8 cm, and an inner diameter of 4 cm. A 10 N force is applied to the other diameter as shown, and a 5 N force is applied to the inner diameter as shown. The pulley is mounted on fixed axle. System: pulley

a. Complete the force diagram for the pulley. (Add to the picture provided.)

b. Complete the force organizer for the pulley. (This requires first setting up an x-y coordinate system and a rotational coordinate system.)

$\vec{F}$	F_x	F_y	τ
$\vec T_1$	0	$-T_1$	$+r_1T_1 = (4cm)(10N)$
$\vec T_2$	0	$-T_2$	$-r_2T_2 = (2cm)(5N)$
$\vec F_G$	0	$-mg$	0 (b/c r=0)
$\vec F_{axle}$	0	$+F_{axle}$	0 (b/c r=0)

c. The pulley is not translating because its axle is fixed. Calculate the force exerted by the axle on the pulley (magnitude and direction). $\Sigma F_y = 0$ on pulley

$F_{axle} - T_1 - T_2 - mg = 0$

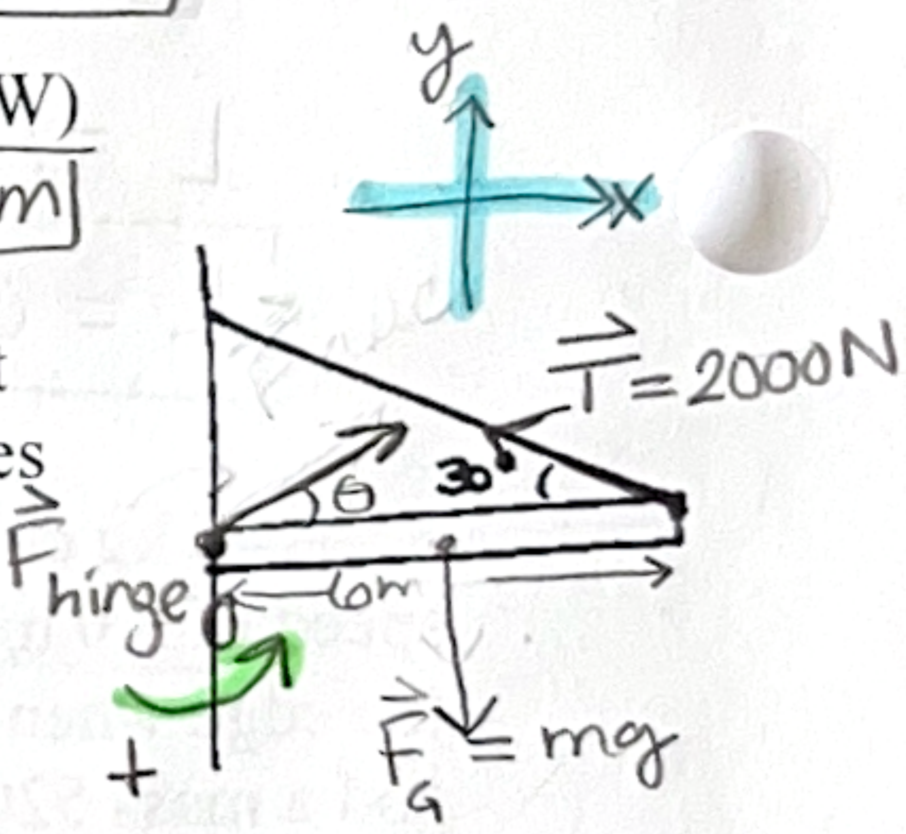
$F_{axle} = T_1 + T_2 + mg$

$= 10N + 5N + (.3kg)(10 N/kg)$

$= \boxed{18 N, \text{up}}$

d. Calculate the sum of the torques on the pulley (include direction as CW or CCW)

$\Sigma\tau = r_1T_1 - r_2T_2 = 40\ N\cdot cm - 10\ N\cdot cm = \boxed{30\ N\cdot cm} = \boxed{.3\ Nm}$



5. A wood beam of length 6 m and mass 200 kg is attached to a wall by a hinge at one end and a rope at the other end. The tension in the rope is 2000 N and it makes an angle of 30° with the beam as shown in the picture. The beam remains at rest. System: wood beam

a. Is the beam in static equilibrium? yes

Is the sum of the forces on the beam equal to zero? yes

Is the sum of the torques on the beam equal to zero? yes

b. Draw a force diagram on the picture provided. (*Hint: There should be 3 force vectors. Do your force vectors add to zero?*) The force of the hinge has to be as shown so that $\Sigma\vec F = 0$

c. Complete the force organizer for the system.

$\vec{F}$	F_x	F_y	τ
$\vec T$	$T_x = -T\cos 30$	$T_y = +T\sin 30$	$\tau_{rope} = (L)(T)\sin 30$
$\vec F_G$	$F_{Gx} = 0$	$F_{Gy} = -mg$	$\tau_{F_G} = -(\frac{L}{2})mg$
$\vec F_H$	$F_{Hx} = F_H\cos\theta$	$F_{Hy} = F_H\sin\theta$	$\tau_H = 0$ (since r=0)

d. Calculate the sum of the torques exerted on the beam about the hinge. Show work.

$\Sigma\tau = LT\sin 30 - (\frac{L}{2})mg$

$= (6m)(2000N)\sin 30 - (\frac{6m}{2})(200kg)(10 N/kg) = 6000\ Nm - 6000\ Nm = \boxed{0}$

e. Calculate the force exerted by the hinge on the beam (magnitude & direction). Show work.

$\Sigma F_y = 0$

$T\sin 30 + F_H\sin\theta - mg = 0$

$(2000N)\sin 30 + F_H\sin\theta - (200kg)(10 N/kg) = 0$

$F_H\sin\theta = 1000$

See next page

$\Sigma F_x = 0$

$F_H\cos\theta - T\cos 30 = 0$

$F_H\cos\theta = (2000N)\cos 30 = 1732N$

$F_H\cos\theta = 1732N$

method2

Divide the two equations:

$$\frac{\cancel{F_H}\sin\theta}{\cancel{F_H}\cos\theta} = \frac{1000N}{1732N}$$

$\tan\theta = .5774$

$\theta = 30^\circ$

Then use either equation to find F_H:

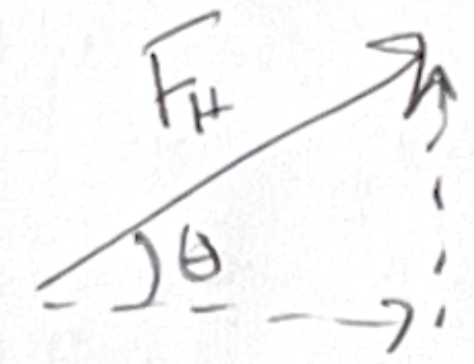
$F_H \sin\theta = 1000N$

$F_H \sin 30^\circ = 1000N$

$F_H = 2000N$

$\boxed{\vec{F}_H = 2000N \text{ at } 30^\circ}$

method 1



→ $F_H \sin\theta$ is just this y-component, so it is 1000N

→ $F_H \cos\theta$ is just this x-component, so it is 1732N

use pythagorean theorem to find F_H and θ:

$F_H = \sqrt{F_x^2 + F_y^2}$

$F_H = \sqrt{(1000N)^2 + (1732N)^2}$

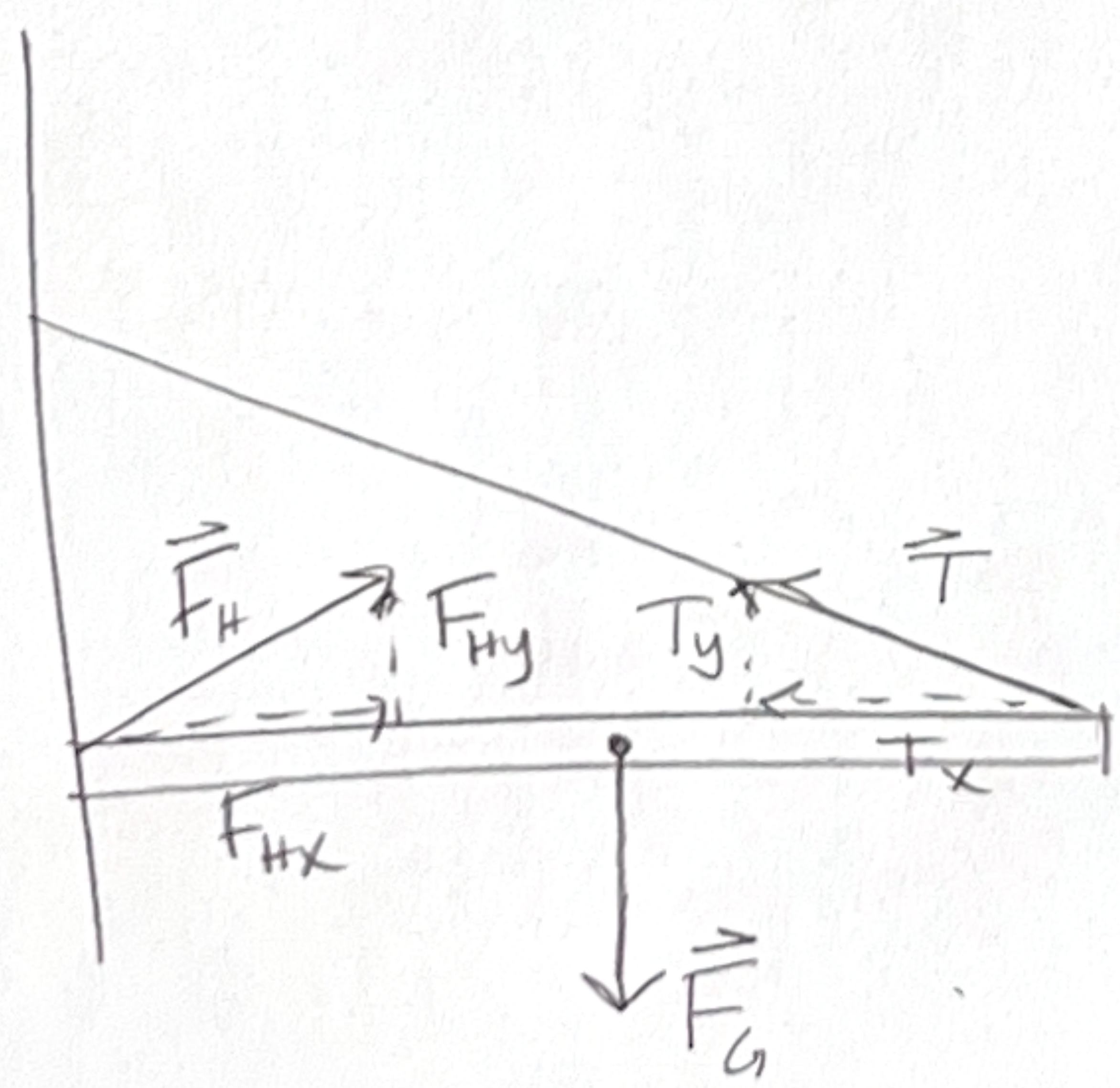
$F_H = 2000N$

$\tan\theta = \frac{1000N}{1732N}$

$\theta = 30^\circ$

$\boxed{\vec{F}_H = 2000N \text{ at } 30^\circ}$

method3



Recognize that F_H and T have to have the same x-component magnitude b/c they have to cancel out.

Recognize that T_y and F_{Hy} have to be the same because F_G is halfway between them.

∴ $F_H = T$, and they are at the same angle since their components are equal.