18

Name: ____________
Date: ____________

Practice: Angular (Momentum and Impulse)

1. Review Linear Momentum

What is linear momentum? The linear momentum of a point particle, or the center of mass of a rigid body, is $\vec{p} = m\vec{v}$. Linear momentum is a vector, and it has the same direction as the translational velocity $\vec{v}$ of the point particle or the center of mass of the rigid body.

What makes linear momentum change? We can find out what makes the momentum change by taking the derivative of the linear momentum equation.

$\vec{p} = m\vec{v}$	Definition of linear momentum
$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$	Take derivative of both sides
$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$	Mass is constant, so it can come out front
$\frac{d\vec{p}}{dt} = m\vec{a}$	The rate of change of velocity is the acceleration
$\frac{d\vec{p}}{dt} = \Sigma\vec{F}$	The product of mass and acceleration is equal to the sum of the forces

This result tells us that the rate of change of momentum, $\frac{d\vec{p}}{dt}$, is equal to the $\Sigma \vec{F}$ on the object. If $\frac{d\vec{p}}{dt} = 0$, what is happening to the linear momentum of the system? not changing (Constant). The sum of the forces is the slope of a p vs. t graph. (because $\Sigma\vec{F} = \frac{dp}{dt}$)

How does the change in momentum $\Delta\vec{p}$ relate to the external forces on the system? We can find out what $\Delta\vec{p}$ is equal to by integrating the previous result.

$\frac{d\vec{p}}{dt} = \Sigma\vec{F}$	Start with the relation derived above
$d\vec{p} = \Sigma\vec{F}dt$	Multiply both sides by dt
$d\vec{p} = (\vec{F}_1 + \vec{F}_2 + \cdots)dt$	Expand out the sum of the forces
$d\vec{p} = \vec{F}_1 dt + \vec{F}_2 dt + \cdots$	Distribute the dt
$\int_{p_i}^{p_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}_1 dt + \int_{t_i}^{t_f} \vec{F}_2 dt + \cdots$	Take the integral of both sides
$\Delta\vec{p} = \Sigma\vec{J}$	Do the integration; Define $\vec{J}$

$\vec{J} = \int_{t_i}^{t_f} \vec{F}dt$ is called the **(linear) impulse**, and we can see that an impulse occurs when an external force acts on the system for some interval of time. The direction of the impulse is the same as the direction of the $\vec{F}$. The impulse is the area of a $\vec{F}$ vs. t graph.

The result $\Delta\vec{p} = \Sigma\vec{J}$ means that the change in momentum of the system, $\Delta\vec{p}$, is equal to the sum of the impulses on the system $\Sigma\vec{J}$. This is the **law of conservation of momentum**: $\Delta\vec{p} = \Sigma\vec{J}$. We will usually use it in this form: $\vec{p}_i + \Sigma\vec{J} = \vec{p}_f$. Linear momentum is *conserved* in all interactions. When will the linear momentum of a system stay *constant*? When $\Sigma\vec{J} = 0$, when there is no net impulse on the system.

2. Angular Momentum for a Rigid Body

What is angular momentum for a rigid body? The angular momentum of a rigid body is $\vec{L} = I\vec{\omega}$. Angular momentum is a vector, and it has the same direction as the angular velocity $\vec{\omega}$ of the rigid body.

What makes angular momentum change? We can find out what makes the angular momentum change by taking the derivative of the angular momentum equation. Fill in the blanks:

$\vec{L} = I\vec{\omega}$	Definition of angular momentum
$\frac{d\vec{L}}{dt} = \frac{d}{dt}(I\vec{\omega})$	Take the derivative of both sides
$\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt}$	Pull the constant I out front
$\frac{d\vec{L}}{dt} = I\vec{\alpha}$	Since $\frac{d\vec{\omega}}{dt} = \alpha$, replace it with α
$\frac{d\vec{L}}{dt} = \Sigma\vec{\tau}$	Since $\alpha = \frac{\Sigma\tau}{I}$, replace $I\alpha$ with $\Sigma\tau$

This result tells us that the rate of change of angular momentum, $\frac{d\vec{L}}{dt}$, is equal to the $\Sigma\vec{\tau}$ on the object. If $\frac{d\vec{L}}{dt} = 0$, what is happening to the angular momentum of the system? not changing (constant). The sum of the torques is the slope of a L vs. t graph.

How does the change in momentum $\Delta\vec{L}$ relate to the external forces on the system? We can find out what $\Delta\vec{p}$ is equal to by integrating the previous result.

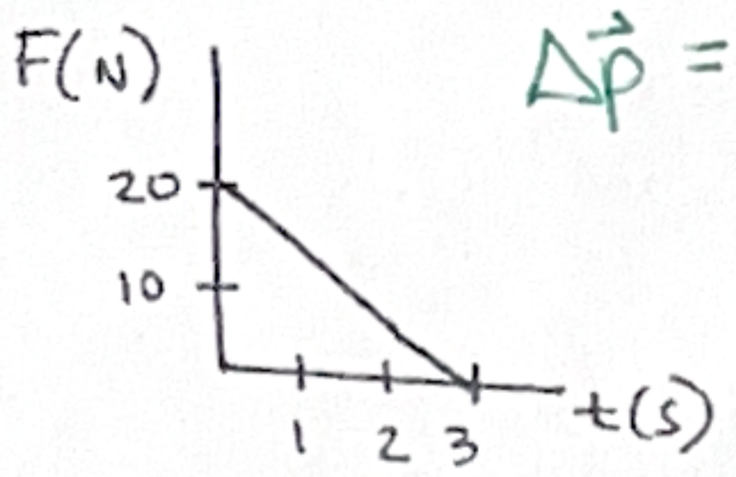
$\frac{d\vec{L}}{dt} = \Sigma\vec{\tau}$	Start with the relation derived above
$d\vec{L} = \Sigma\vec{\tau}dt$	Multiply both sides by dt
$d\vec{L} = (\vec{\tau}_1 + \vec{\tau}_2 + \cdots)dt$	Expand $\Sigma\vec{\tau}$ to show individual torques
$d\vec{L} = \vec{\tau}_1 dt + \vec{\tau}_2 dt + \cdots$	Distribute the dt
$\int_{L_i}^{L_f} d\vec{L} = \int_{t_i}^{t_f} \vec{\tau}_1 dt + \int_{t_i}^{t_f} \vec{\tau}_2 dt + \cdots$	Take integral of both sides
$\Delta\vec{L} = \Sigma\vec{K}$, where $\vec{K} = \int_{t_i}^{t_f} \vec{\tau} dt$	Complete integration, Define $\vec{K}$

$\vec{K} = \int_{t_i}^{t_f} \vec{\tau} dt$ is called the **angular impulse**, and we can see that it occurs when an external torque acts on the system for some interval of time. The direction of the angular impulse is the same as the direction of the $\vec{\tau}$. The angular impulse is the area of a τ vs. t graph.

The result $\Delta\vec{L} = \Sigma\vec{K}$ means that the change in momentum of the system, $\Delta\vec{L}$, is equal to the sum of the angular impulses on the system $\Sigma\vec{K}$. This is the **law of conservation of** angular **momentum**: $\Delta\vec{L} = \Sigma\vec{K}$. We will usually use it in this form: $\vec{L}_i + \Sigma\vec{K} = \vec{L}_f$. Angular momentum is *conserved* in all interactions. When will the angular momentum of a system stay *constant*? When $\Sigma\vec{K} = 0$ (When the net angular impulse is 0.)

3. Interpreting Graphs (It is not necessary to show work.)

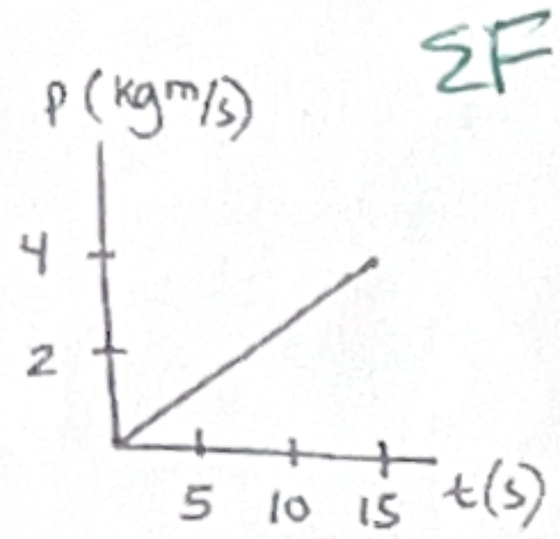
a. This graph shows the force on an object as a function of time. Calculate the change in momentum of the object from 0 to 3 s.



$\Delta\vec{p} = \Sigma\vec{J}$ = area of F vs. t graph

$= \boxed{30\ \text{kgm/s}}$

b. This graph shows the linear momentum of an object as a function of time. Calculate the net force on the object during the time interval from 0s to 15 s.

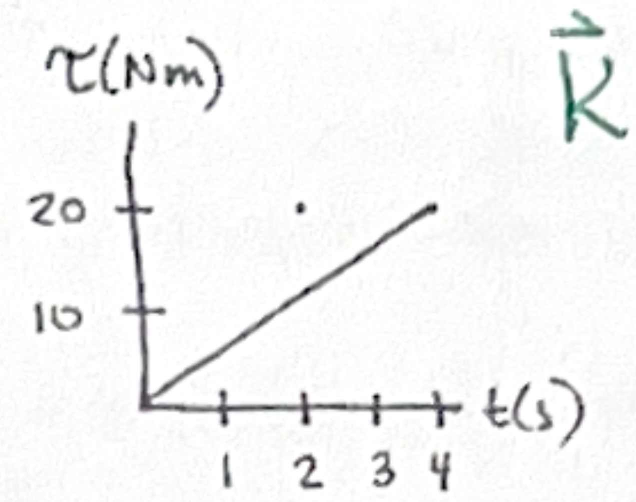


$\Sigma F = \frac{dp}{dt}$ = slope of p vs. t graph

$= 0.27\ \text{kgm/s}^2$

$= \boxed{0.27\ \text{N}}$

c. This graph shows the torque on a system as a function of time. Calculate the angular impulse exerted on the system from 1s to 4 s.

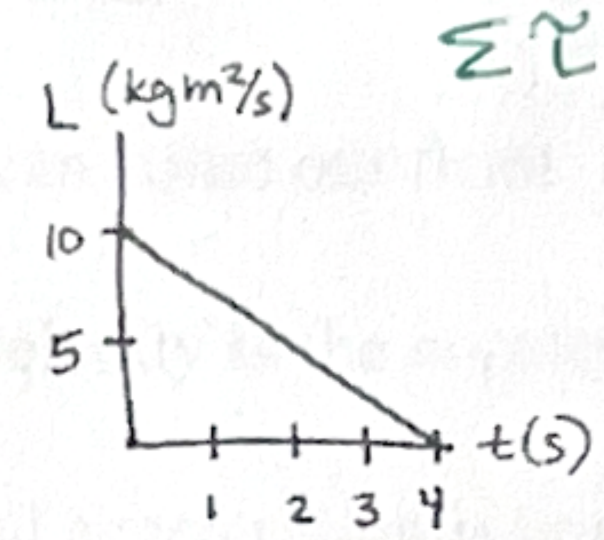


$\vec{K} = \int \vec{\tau}\, dt$

= area of τ vs. t graph

$= \boxed{40\ \text{Nms}}$

d. This graph shows the angular momentum of an system as a function of time. Calculate the net torque on the system from 0s to 4 s.



$\Sigma\tau = \frac{dL}{dt}$ = slope of L vs t graph

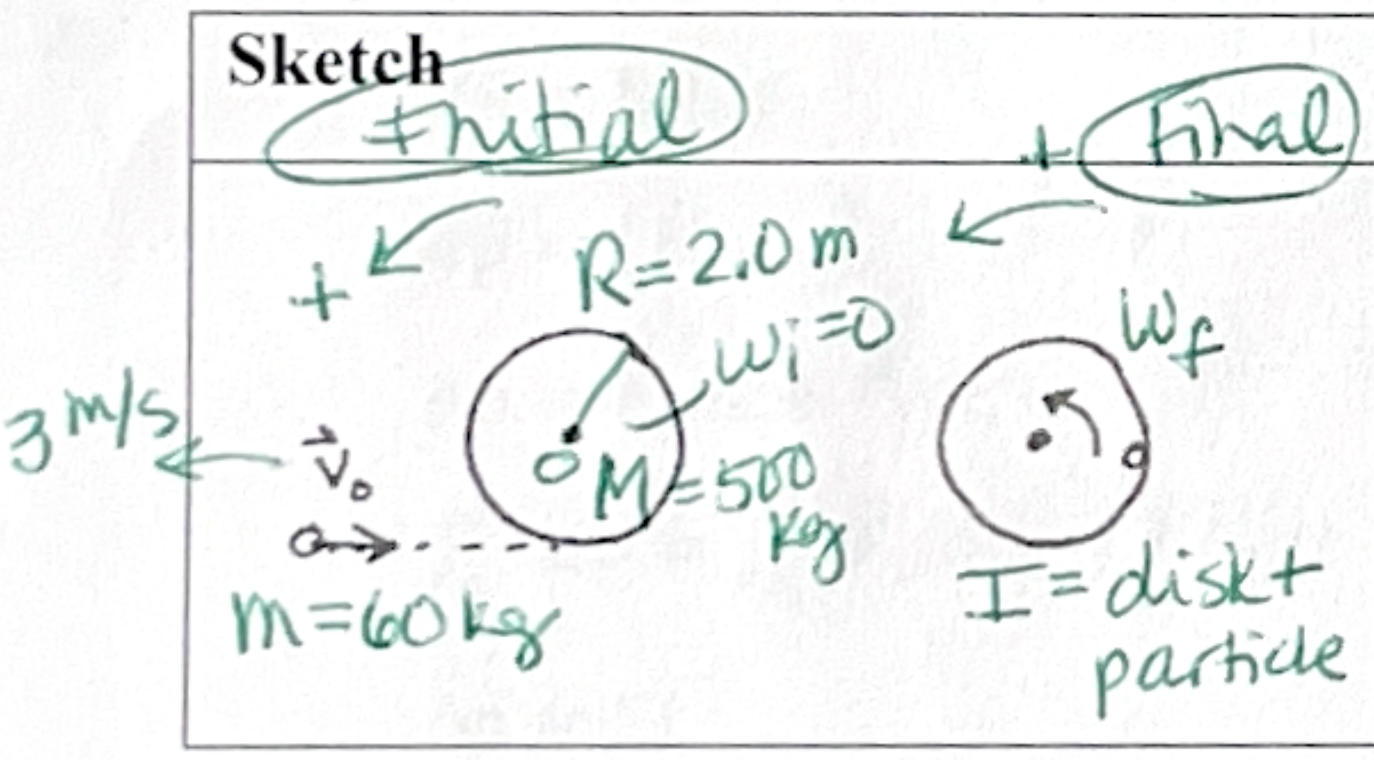
$= -2.5\ \text{kgm}^2/\text{s}^2$

$= \boxed{-2.5\ \text{Nm}}$

4. Problem Solving (Show work)

A child ~~is~~ runs toward a merry-go-round that is initially at rest and jumps onto the edge. The child has mass 60 kg, and is running with a speed of 3 m/s. The merry-go-round has a mass of 600 kg and radius 2.0 m, and we will model it as rotating on frictionless bearings. Find the angular speed of the merry-go-round and child after the child jumps on.

a. Complete the sketch and define your system. Label initial and final times, point O, coordinate system, define variables. Then complete the last three columns of the chart symbolically.

Sketch	System	Initial L?	Ext Torques? Angular Impulses?	Final L?
	Child merry-go-round axle, earth	$L_i = L_{child}$ $= r_\perp mv$, + direction, $L_i = +Rmv$	Ext forces are F_G, axle. Neither one causes a torque, so $\Sigma\vec{K} = 0$	$L_f = L$ of composite object. $I = \frac{1}{2}MR^2 + mR^2$ $L_f = I\omega_f$

b. Write a conservation of angular momentum equation for the situation and solve for the unknown.

$$\vec{L}_i + \Sigma\vec{K} = \vec{L}_f$$

$$+Rmv + 0 = \left(\tfrac{1}{2}MR^2 + mR^2\right)\omega_f$$

$$Rmv = R^2\left(\tfrac{1}{2}M + m\right)\omega_f$$

$$\frac{mv}{R\left(\frac{1}{2}M + m\right)} = \omega_f$$

$$\frac{(60\ \text{kg})(3\ \text{m/s})}{(2.0\ \text{m})\left[\frac{1}{2}(500\ \text{kg}) + 60\ \text{kg}\right]} = \omega_f$$

$$= \frac{180\ \text{kgm/s}}{620\ \text{kgm}}$$

$$= \boxed{0.29\ \text{rad/s}}$$