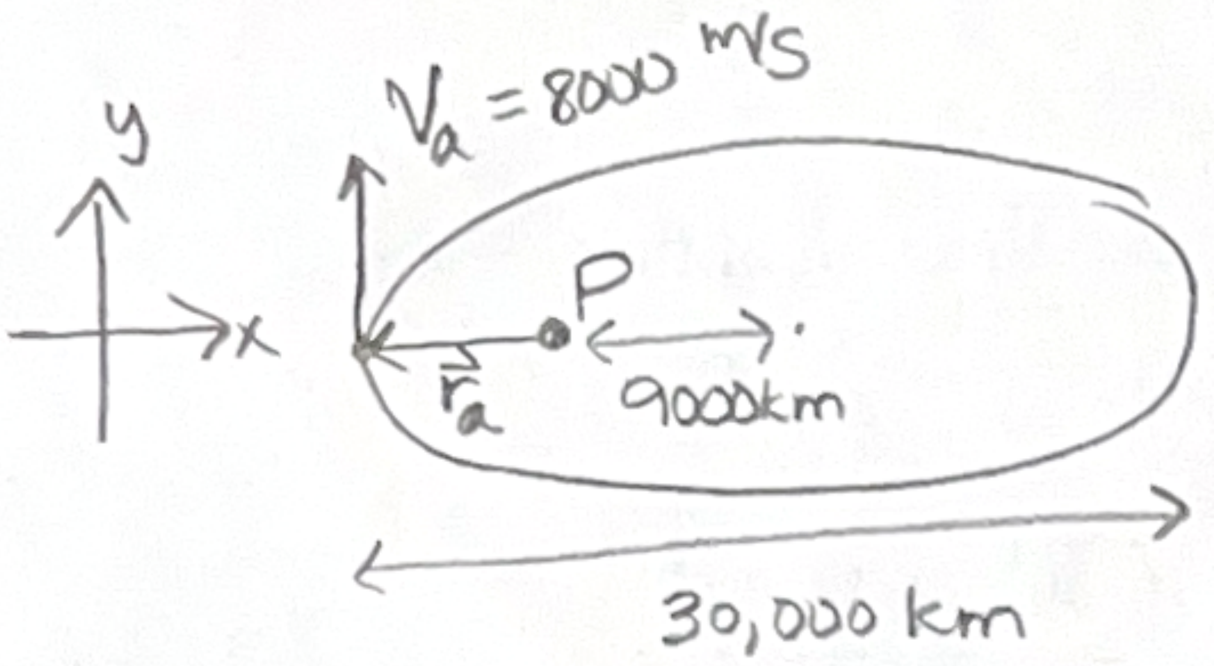
Name: ____________
Date: ____________

4

Solutions posted

Practice: Rotational Kinematics

1) p. 334 #77. Don't do the problem as stated. Instead, use the given informationin the problem statement and the picture to find the angular momentum vector of the satellite about the Planet at point *a* (in unit vector notation), if the mass of the satellite is 700 kg. Include a sketch that shows the position vector and the velocity vector you are using in your calculation.



$r_a = 15{,}000\text{ km} - 9000\text{ km} = 6000\text{ km} = 6\times10^{6}\text{ m}$

$V_a = 8000$ m/s

$m = 700$ kg

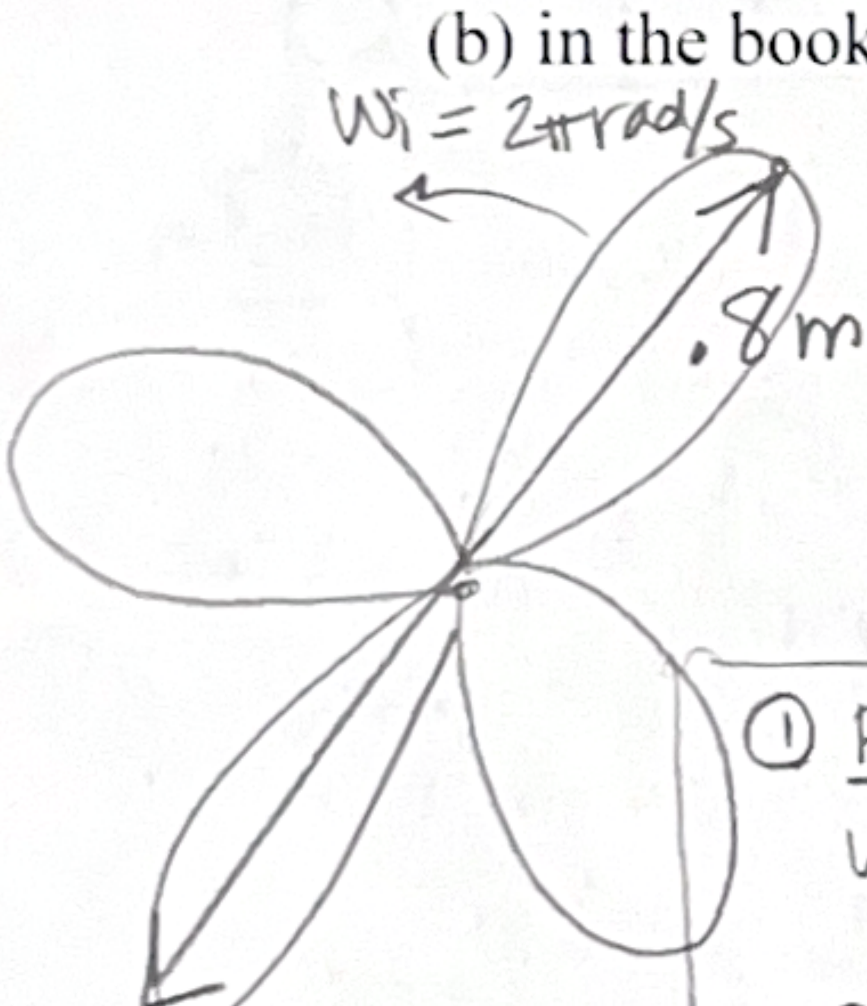
$L = r_\perp m v$

$L = (6\times10^{6}\text{ m})(700\text{ kg})(8000\text{ m/s})$

$L = 3.36\times10^{13}\ \text{kg}\cdot\text{m}^2/\text{s}$ ← magnitude

$\boxed{\vec{L} = -3.36\times10^{13}\ \frac{\text{kg m}^2}{\text{s}}\ \hat{k}}$ ← vector

2) p.330 # 3. First calculate the angular acceleration of the fan as it is stopping. Then do (a) and (b) in the book. (Tip: Convert rpm to rad/s)



diameter of fan = 0.8m

$\omega_i = 60\text{ rpm} = 60\ \frac{\text{rev}}{\text{min}}\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 2\pi$ rad/s

$\omega_f = 0$

$\Delta t = 25$ s

① Find α

$\omega_f = \omega_i + \alpha\Delta t$

$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$

$\alpha = \frac{0 - 2\pi\text{ rad/s}}{25\text{ s}}$

$\boxed{\alpha = -.25\text{ rad/s}^2}$

ⓐ Speed of tip: That is V_t at $r = .4$ m at $\Delta t = 10$ s

$V_t = \omega r \rightarrow$ So I need to know ω_f at $\Delta t = 10$ s. The angular acceleration α is $-.25$ rad/s² which I assume is constant.

$\omega_f = ?$

$\omega_i = 2\pi$ rad/s

$\Delta t = 10$ s

$\alpha = ?$

$\omega_f = \omega_i + \alpha\Delta t$

$\omega_f = 2\pi\ \frac{\text{rad}}{\text{s}} + (-.25\frac{\text{rad}}{\text{s}^2})(10\text{s})$

$\omega_f = 3.8$ rad/s

$V_t = \omega r$

$V_t = (3.8\frac{\text{rad}}{\text{s}})(.40\text{m})$

$\boxed{V_t = 1.5\text{ m/s}}$

ⓑ Find $\Delta\theta$

$\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2$

$\Delta\theta = (2\pi\ \frac{\text{rad}}{\text{s}})(25\text{s}) + \frac{1}{2}(-.25\frac{\text{rad}}{\text{s}^2})(25\text{s})^2$

$\Delta\theta = 79$ rad

Convert to Rev

$\hookrightarrow 79\text{ rad}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 12.6$ rev

3) The position as a function of time for a pulley of mass 3 kg and radius 0.5 m is $\theta = 2 + 5t + 10t^3$, where the angle is in radians and time is in seconds.

a) What are the units of the 2 in this equation? What does it represent about the motion?

radians, represents initial θ at $t=0$.

b) What are the units of the 5 in this equation?

rad/s

c) What are the units of the 10 in this equation?

rad/s^3

d) Derive an equation for the angular acceleration of the pulley as a function of time.

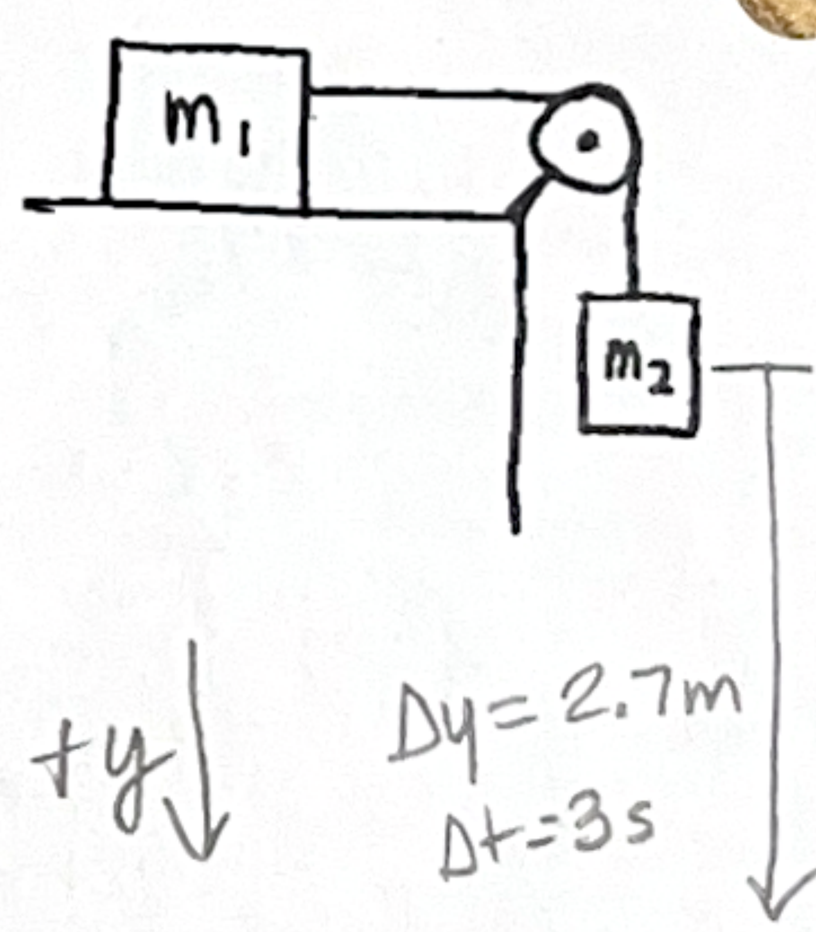
$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}(2+5t+10t^3) = 5 + 30t^2$$

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(5+30t^2) = 60t \longrightarrow \boxed{\alpha(t) = 60t}$$

e) Find the angular acceleration of the pulley at t = 3.0 seconds.

$$\alpha(3.0s) = 60(3s) = \boxed{180\ \text{rad}/s^2}$$

4) A modified Atwood's machine is released from rest. The pulley has mass M and radius of 5 cm. The hanging mass moves 2.7 meter in 3 seconds.



a. What is the SAME for the hanging mass and the pulley?

b. Calculate the acceleration of the hanging mass

$\Delta y = 2.7m$
$V_{iy} = 0$
$V_{fy} = ?$
$a_y = ?$
$\Delta t = 3s$

$$\Delta y = \cancel{V_{iy}}^{0}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$a_y = \frac{2\Delta y}{\Delta t^2}$$

$$a_y = \frac{2(2.7m)}{(3s)^2} = \boxed{.6\ m/s^2}$$

c. Calculate the angular acceleration of the pulley.

block
accel is a_m

pulley
$a_t = \alpha r$

The connection between the block and the pulley is the acceleration of the falling mass a_m equals the tangential acceleration of the edge of the pulley. $a_m = a_t = a$ $\therefore$ $a = \alpha r$

$$(.6\ m/s^2) = \alpha(.05m)$$

$$\boxed{\alpha = 12\ \text{rad}/s^2}$$