Name: ____________

Date: ____________

SOLUTIONS ARE POSTED

Practice – Angular Momentum of a Particle

1. Write these facts into your physics facts booklet:

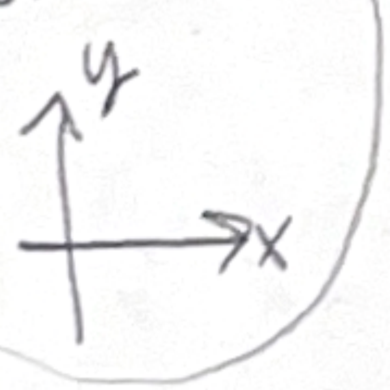
- The angular momentum $\vec{L}$ of a particle about point O is $\vec{L} = \vec{r} \times \vec{p}$, which is the cross product of its position vector $\vec{r}$ and its linear velocity vector $\vec{v}$.
- The magnitude of the angular momentum is $L = (r)(mv)sin\phi$ where the angle ϕ is the angle between the two vectors when they are drawn with their tails together.
- The magnitude of the angular momentum can be found two ways:
  - Use the component of $\vec{v}$ (or $\vec{p}$) that is ⊥ to $\vec{r}$: $\boldsymbol{L = (r)(mv_{\perp})}$
  - Use the component $\vec{r}$ that is ⊥ to $\vec{v}$ (or $\vec{p}$): $\boldsymbol{L = (r_{\perp})(mv)}$
- The angular momentum can be described as clockwise or counterclockwise, but its direction as a vector is perpendicular to the plane of motion according to the right- hand rule.

2. What is the equation for the angular momentum vector $\vec{L}$? $\vec{L}=\vec{r}\times\vec{p}$

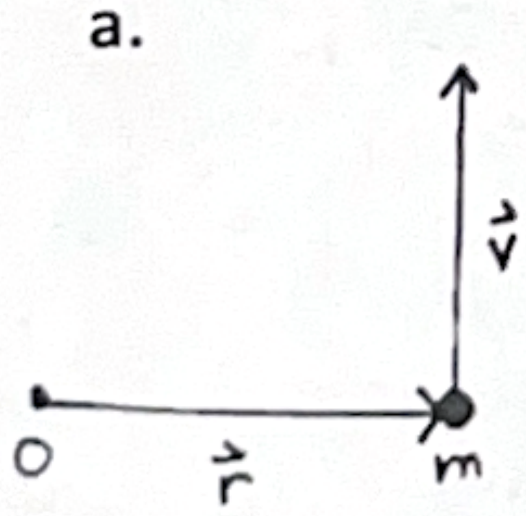
3. What are the two equations for the magnitude of the angular momentum in terms of perpendicular components of the vectors involved? $L=(r)(mV_{\perp})$ and $L=(r_{\perp})(mv)$

4. Calculate the angular momentum vector $\vec{L}$ about point O of particle m below using one of the equations you wrote in (3). Show your work, writing the equation at the top of your calculation, and expressing your answer in unit vector notation. You will need to set up a right-handed coordinate system.

for all:



a.



$m = 2kg$

$V = 5\,m/s$

$r = 0.3m$

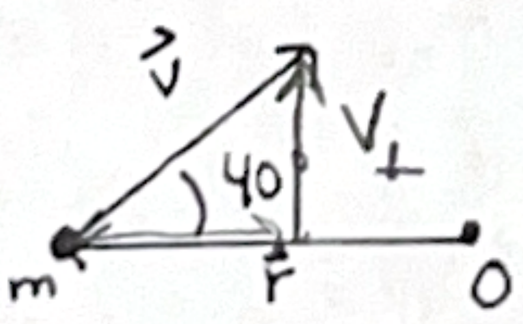
$L=(r_{\perp})(mv)$

$L=(.3m)(2kg)(5\,m/s)$

$L = 3\,kgm^2/s$

$\vec{L} = 3\,kgm^2/s\ \hat{k}$

b.



$m = 2kg$

$V = 5\,m/s$

$r = 0.3m$

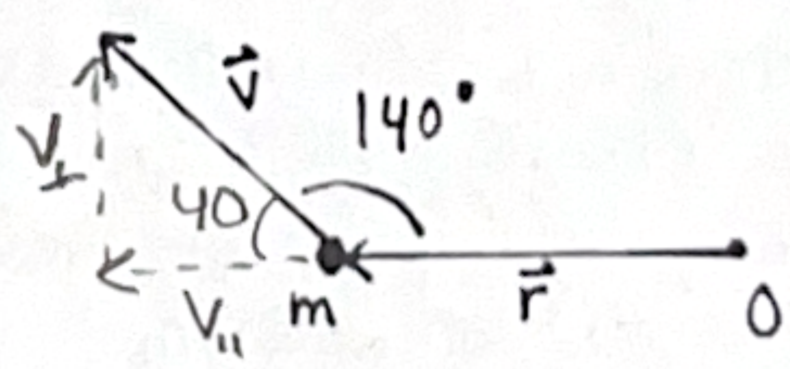
$L=(r)(mV_{\perp})$

$L=(.3m)(2kg)(5\,m/s)(\sin 40)$

$L = 1.9\,kgm^2/s$

$\vec{L} = -1.9\,kgm^2/s\ \hat{k}$

c.



$m = 2kg$

$V = 5\,m/s$

$r = .3m$

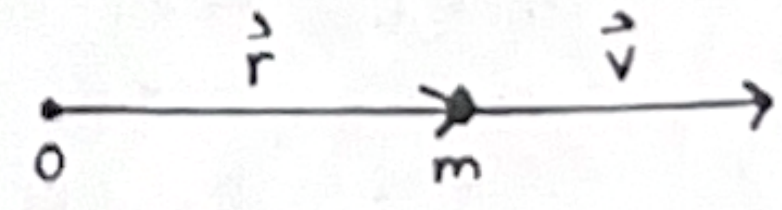
$L=rmV_{\perp}$

$L=(.3m)(2kg)(5\,m/s)(\sin 40)$

$L=1.9\,kgm^2/s$

$\vec{L} = -1.9\,kgm^2/s\ \hat{k}$

d.



$m = 2kg$

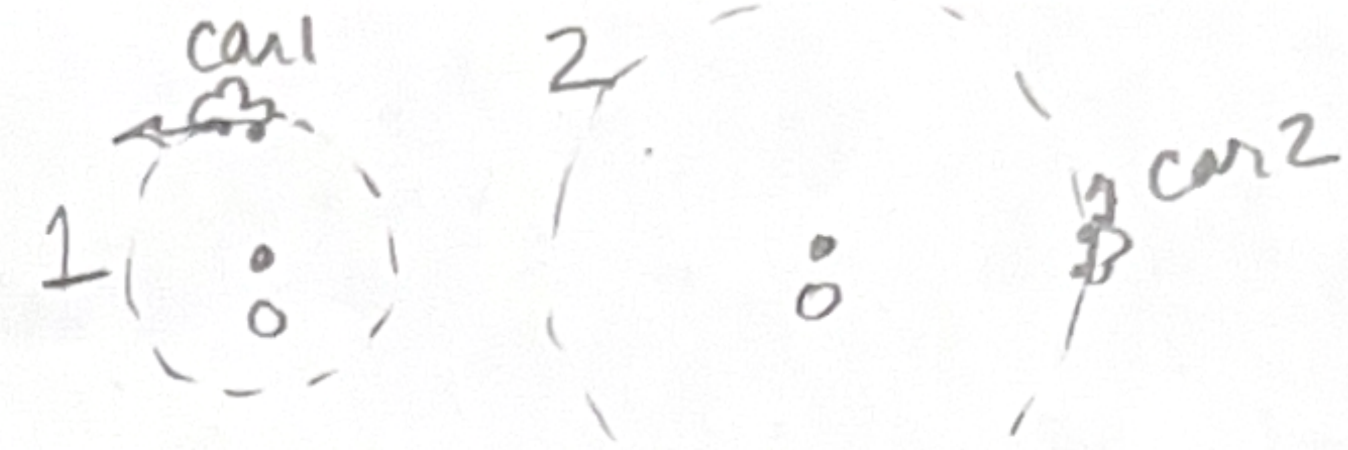
$V = 5\,m/s$

$r = .3m$

$L=rmV_{\perp}$

There is no ⊥ component of $\vec{v}$ relative to $\vec{r}$, so

$\vec{L}=0$



$L_1 = r m v_\perp$

5. Imagine that Circular path 2 has a twice the radius of circular path 1. Car 1 travels on circular path 1. Car 2 travels on circular path 2. Point O is at the center of each circular path. Compare the angular momentum of Car 1 and Car 2 about point O for each scenario by writing >, =, or < in the blank.

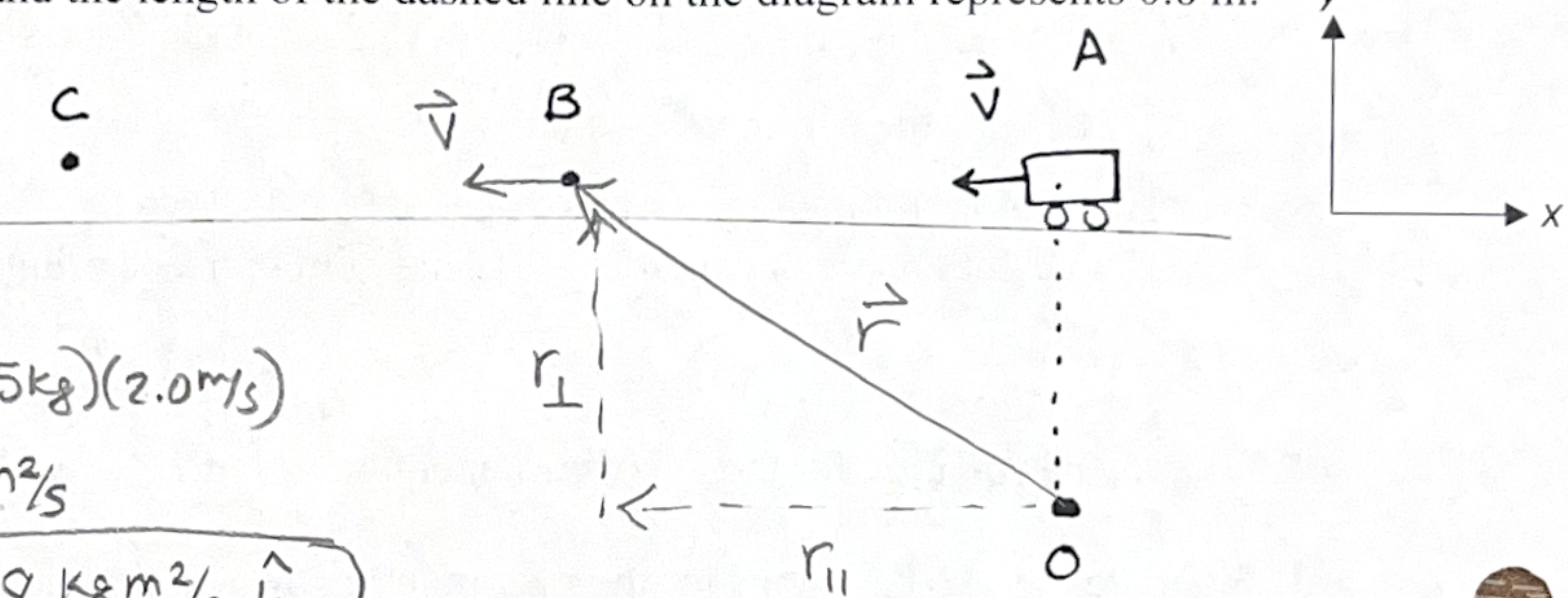
a. Car 1 and Car 2 are identical and they both travel with the same linear speed. L_1 __<__ L_2

b. Both cars have the same linear speed, but Car 2 has half the mass of Car 1. L_1 __=__ L_2

$L_1 = rmv$ $L_2 = (2r)(\frac{1}{2}m)v$ } car 2 has half the mass, same speed, double r

$L_2 = rmv$

6a. A cart is moving with a constant velocity $\vec{v}$ *as shown below*, and point O is off to the side of the line of motion. On the picture, draw the cart's position vector and velocity vector when it is at B. Then calculate the cart's momentum vector $\vec{L}$ about point O when it is at B, if the mass of the cart is 500 g, its speed is 2.0 m/s, and the length of the dashed line on the diagram represents 0.8 m.



$r_\perp = .8m$
$m = .5kg$
$V = 2.0 m/s$

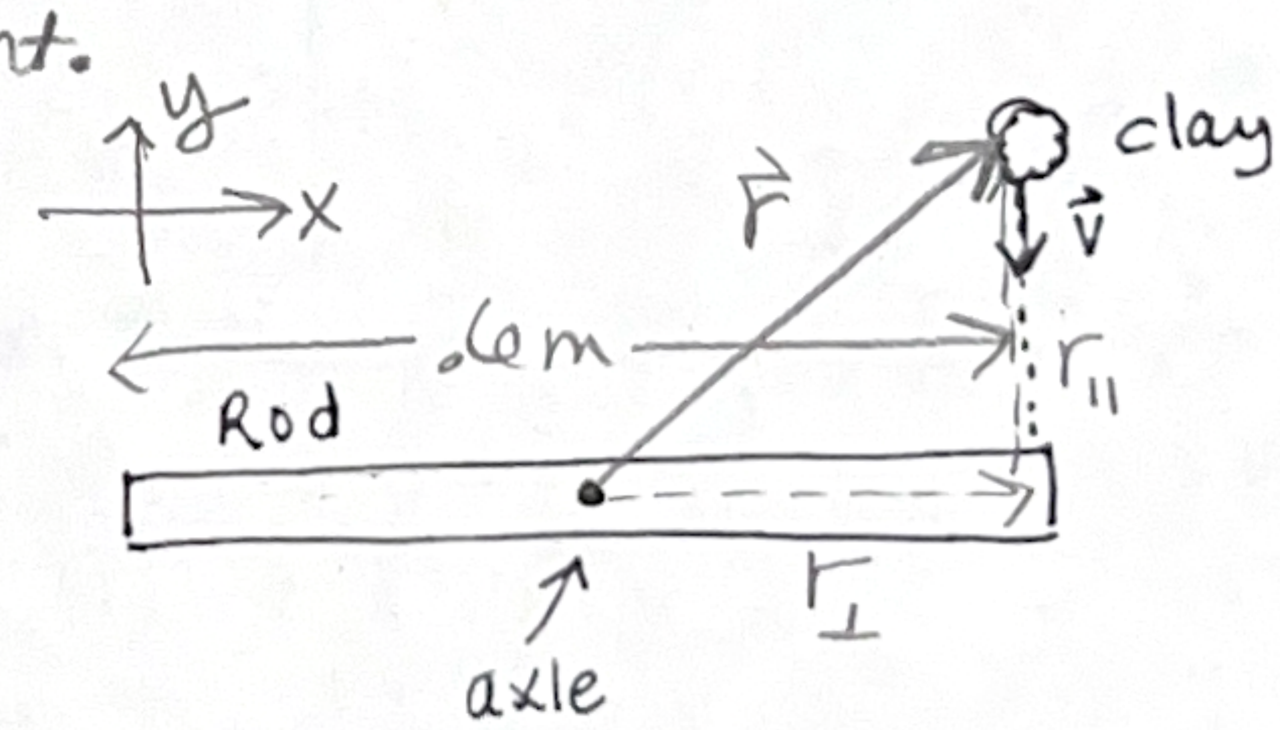
$L = r_\perp mv$
$L = (.8m)(.5kg)(2.0m/s)$
$L = .8\ kgm^2/s$

vector $\vec{L} = .8\ kgm^2/s\ \hat{k}$

6b. We saw in class that the cart's angular momentum is the same at points A, B, and C. How is this possible, since the position vector is larger at points B and C, than it is at A?

The angular momentum (magnitude) only depends on the component of $\vec{r}$ that is $\perp$ to the velocity. For the cart, this $r_\perp$ is always the same. Since $L = r_\perp mv$, and the mass and speed are also constant, L is constant.

7. A uniform horizontal rod of length 0.6 m and mass 3.0 kg is mounted on an axle that passes vertically through its center of mass, as shown in the *aerial-view* diagram. The rod is initially at rest. A blob of clay of mass 0.5 kg is moving as shown with a speed of 2.0 m/s. When it collides with the end of the rod, it will stick, and they will rotate together about the axle.



Find the angular momentum vector $\vec{L}$ of the blob of clay about the axle before it makes impact. (To give the direction of the vector, either describe the direction of the vector relative to the page, or set up a coordinate system and use a unit vector.)

$r_\perp = .3m$
$m = .5kg$
$V = 2.0 m/s$

$L = r_\perp mv$
$L = (.3m)(.5kg)(2.0m/s)$
$L = .3\ kgm^2/s$

$\vec{L} = -.3\ kgm^2/s\ \hat{k}$