Solutions posted

Name: ____________

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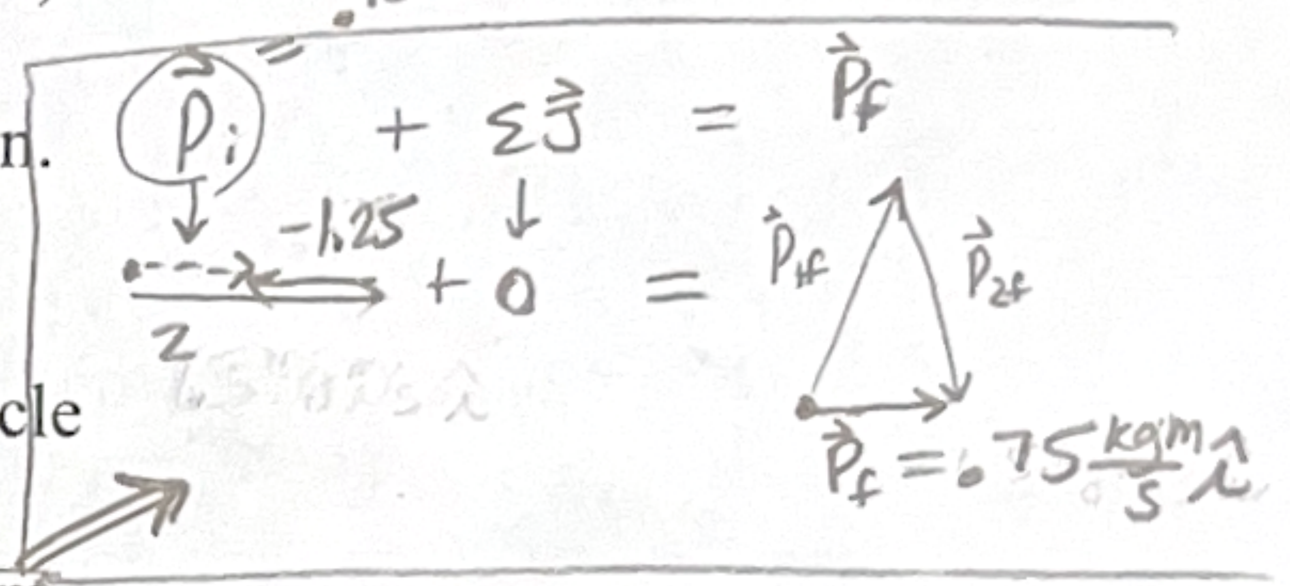
Momentum and Energy

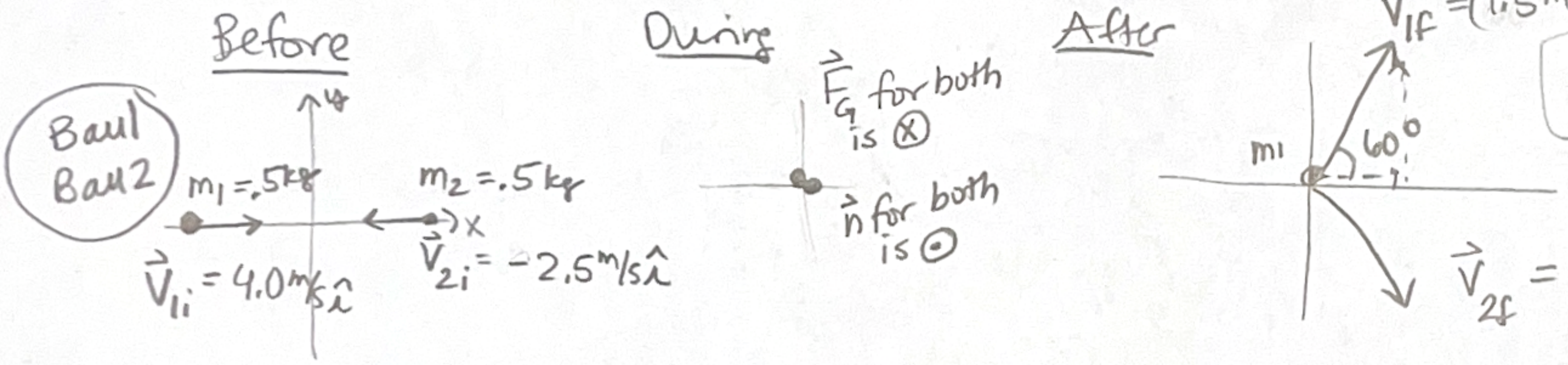
1. Two balls of equal mass of 0.500 kg undergo a glancing collision. Before the collision, ball 1 was moving east with a speed of 4.0 m/s, and ball 2 was moving west with a speed of 2.5 m/s. After the collision, the velocity of ball 1 is (1.5 m/s, 60° north of east).

a. Find the velocity of ball 2 after the collision in unit vector notation.

Include the following:

- *Before*, *during*, *after* sketches, coordinate system, system circle
- External force & impulse analysis
- Calculations and a conservation of momentum vector diagram.





$\vec{V}_{1f} = (1.5 m/s, 60°)$

$\vec{V}_{1f} = 1.5 m/s \cos 60 \hat{\imath} + 1.5 m/s \sin 60 \hat{\jmath}$

$= .75 m/s \hat{\imath} + 1.3 m/s \hat{\jmath}$

Ext forces & Impulses

$\vec{F}_{G1} \to \vec{J}_1$, $\vec{n}_1 \to \vec{J}_2$) cancel

$\vec{F}_{G2} \to \vec{J}_3$, $\vec{n}_2 \to \vec{J}_4$) cancel

$\vec{P}_i + \Sigma\vec{J} = \vec{P}_f$

$m_1\vec{V}_{1i} + m_2\vec{V}_{2i} + 0 = m_1\vec{V}_{1f} + m_2\vec{V}_{2f}$

$(.5kg)(4.0 m/s \hat{\imath}) + (.5kg)(-2.5 m/s \hat{\imath}) = (.5kg)(.75 m/s \hat{\imath} + 1.3 m/s \hat{\jmath}) + (.5kg)(\vec{V}_{2f})$

$4.0 m/s \hat{\imath} - 2.5 m/s \hat{\imath} = .75 m/s \hat{\imath} + 1.3 m/s \hat{\jmath} + \vec{V}_{2f}$

$.75 m/s \hat{\imath} - 1.3 m/s \hat{\jmath} = \vec{V}_{2f}$

b. Calculate the impulse exerted by ball 1 on ball 2 during the collision, expressing your answer in unit vector notation. Include a system circle and a sketch of the vector diagram that represents your calculation.

To find impulse on ball 2, I have to include only Ball 2 in the system so the force + impulse from ball 1 will be external and show up in the equation.

Ball 2

$\vec{P}_i + \Sigma\vec{J} = \vec{P}_f$

$m_2\vec{V}_{2i} + \Sigma\vec{J}_{on\ Ball 2} = m_2\vec{V}_{2f}$

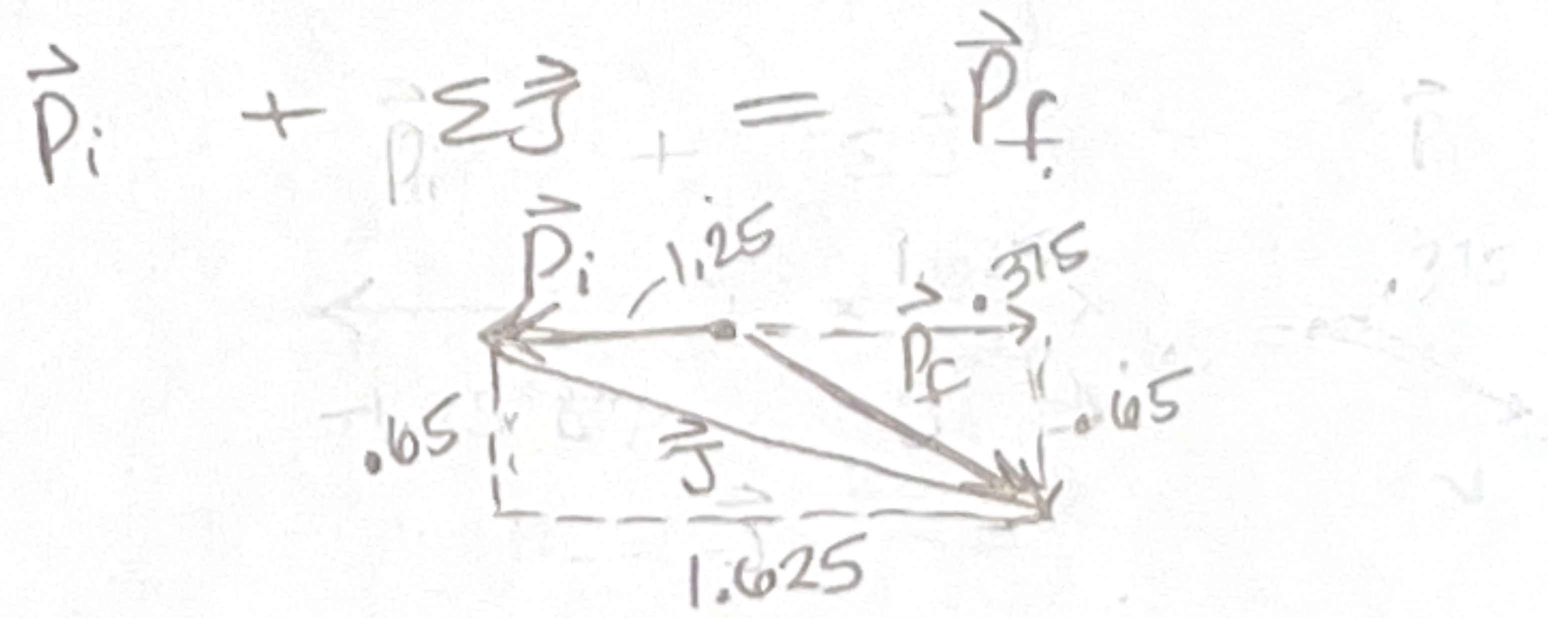
$\Sigma\vec{J} = m_2\vec{V}_{2f} - m_2\vec{V}_{2i}$

$\Sigma\vec{J} = m_2(\vec{V}_{2f} - \vec{V}_{2i})$

$\Sigma\vec{J} = (.5kg)[(.75 m/s \hat{\imath} - 1.3 m/s \hat{\jmath}) - (-2.5 m/s \hat{\imath})]$

$\Sigma\vec{J} = .5kg\,[.75 m/s \hat{\imath} + 2.5 m/s \hat{\imath} - 1.3 m/s \hat{\jmath}]$

$\Sigma\vec{J} = 1.625 Ns \hat{\imath} - .65 Ns \hat{\jmath}$



Since the impulses from $\vec{F}_G$ and $\vec{n}$ cancelled, this is $\vec{J}_{on\ Ball 2}$

2. A satellite (m_s) is initially in a higher circular orbit of radius R_2 around Earth (m_e) Re, when the rockets are fired and its orbit is changed to a lower circular orbit of radius R_1. (R_1 and R_2 are measured from the center of the Earth.)

a. What are each of these in terms of U_G, the potential energy of the satellite-earth system?

The total energy of the satellite-earth system: TE = $\frac{1}{2}U_G$

The kinetic energy of the satellite-earth system: K = $-\frac{1}{2}U_G$ (so $K = -\frac{1}{2}TE$)

b. In which orbit does the satellite-Earth system have (i) the greatest potential energy? R_2 (ii) the greatest kinetic energy? R_1 (iii) the greatest total energy? R_2

c. Determine an expression for (i) the potential energy and (ii) the kinetic energy of the satellite-earth system when the satellite is in the higher orbit in terms of given variables and fundamental constants.

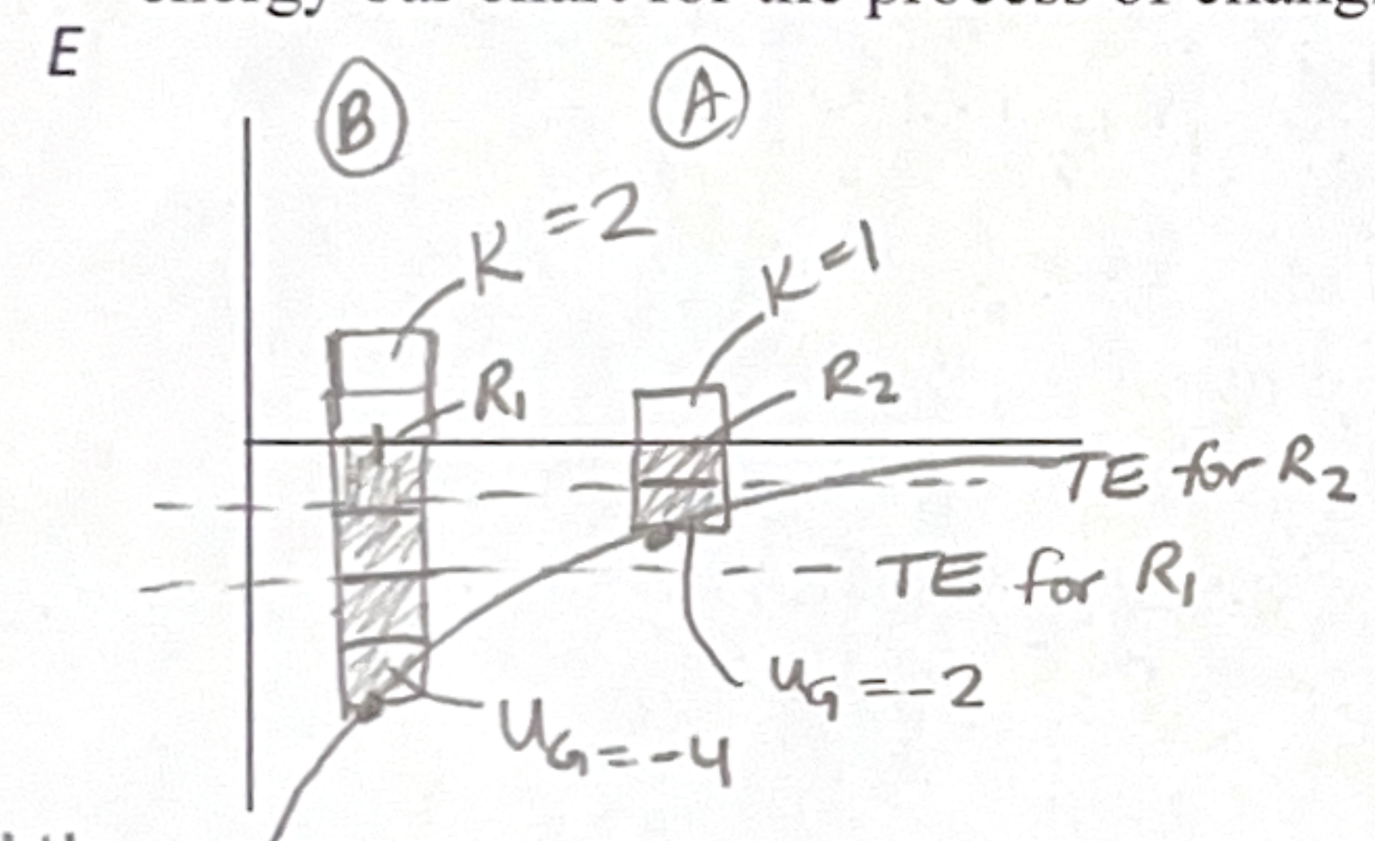
$U_G = -\frac{Gm_1m_2}{R} = \boxed{-\frac{Gm_em_s}{R_2}}$

(iii) total energy: $TE = \frac{1}{2}U_G = \boxed{-\frac{1}{2}\frac{Gm_em_s}{2R_2}}$

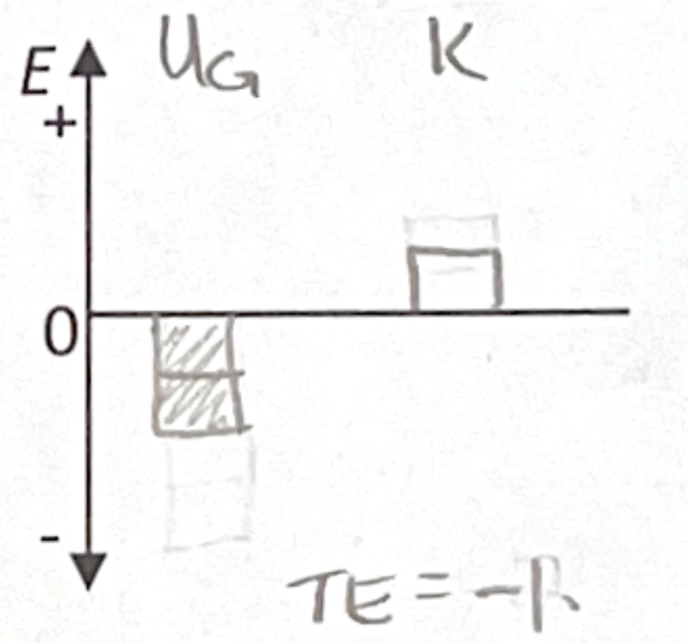
$K = -\frac{1}{2}U_G = -\frac{1}{2}\left(-\frac{Gm_em_s}{R_2}\right) = \boxed{\frac{1}{2}\frac{Gm_em_s}{R_2}}$

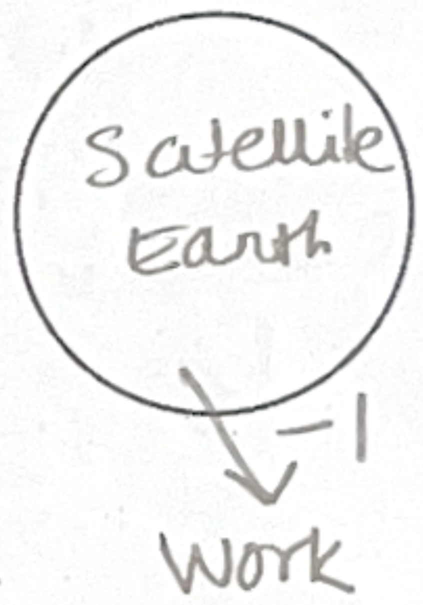
You can also use circular motion to find v and Kinetic energy, but it takes longer

d. Draw a qualitative energy graph showing U_G, K, and TE for both orbits. Draw a matching energy bar chart for the process of changing orbits.

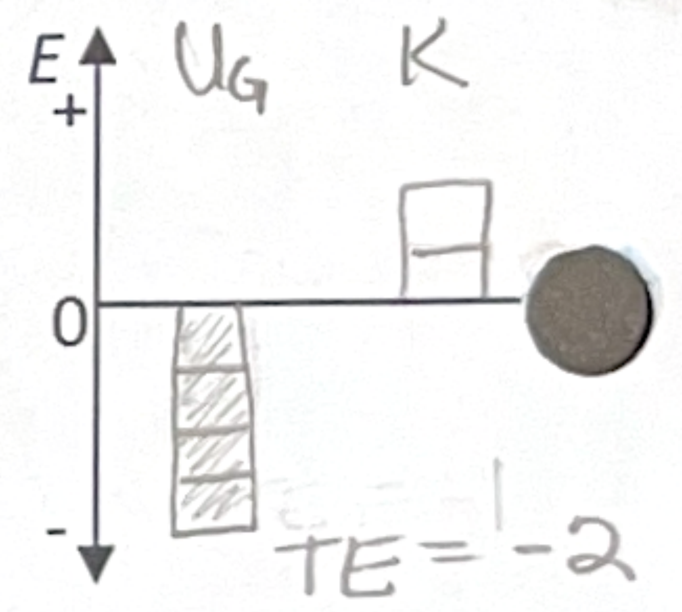


Initial time: Orbit R_2 (A)



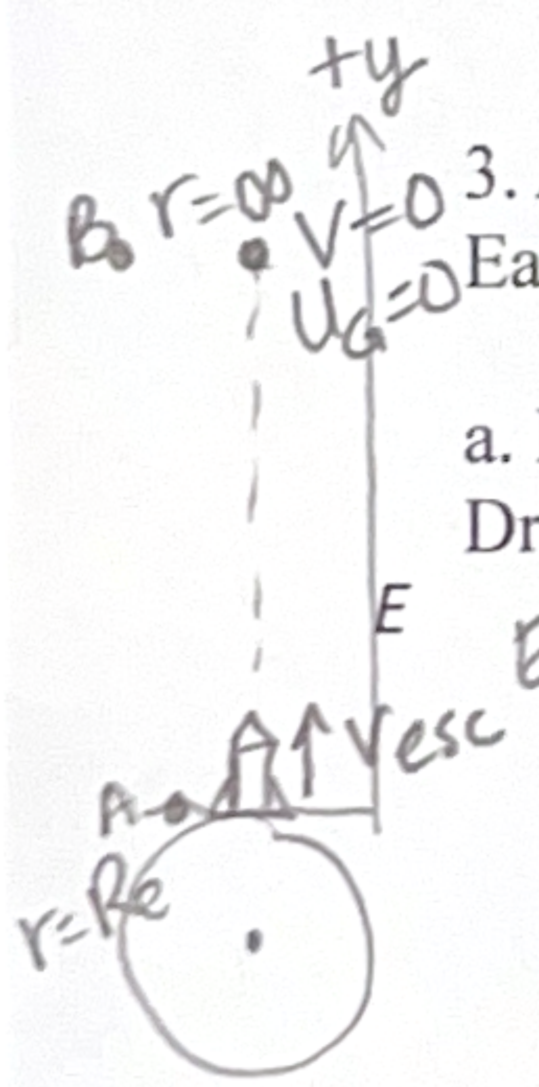


Final time: Orbit R_1 (B)

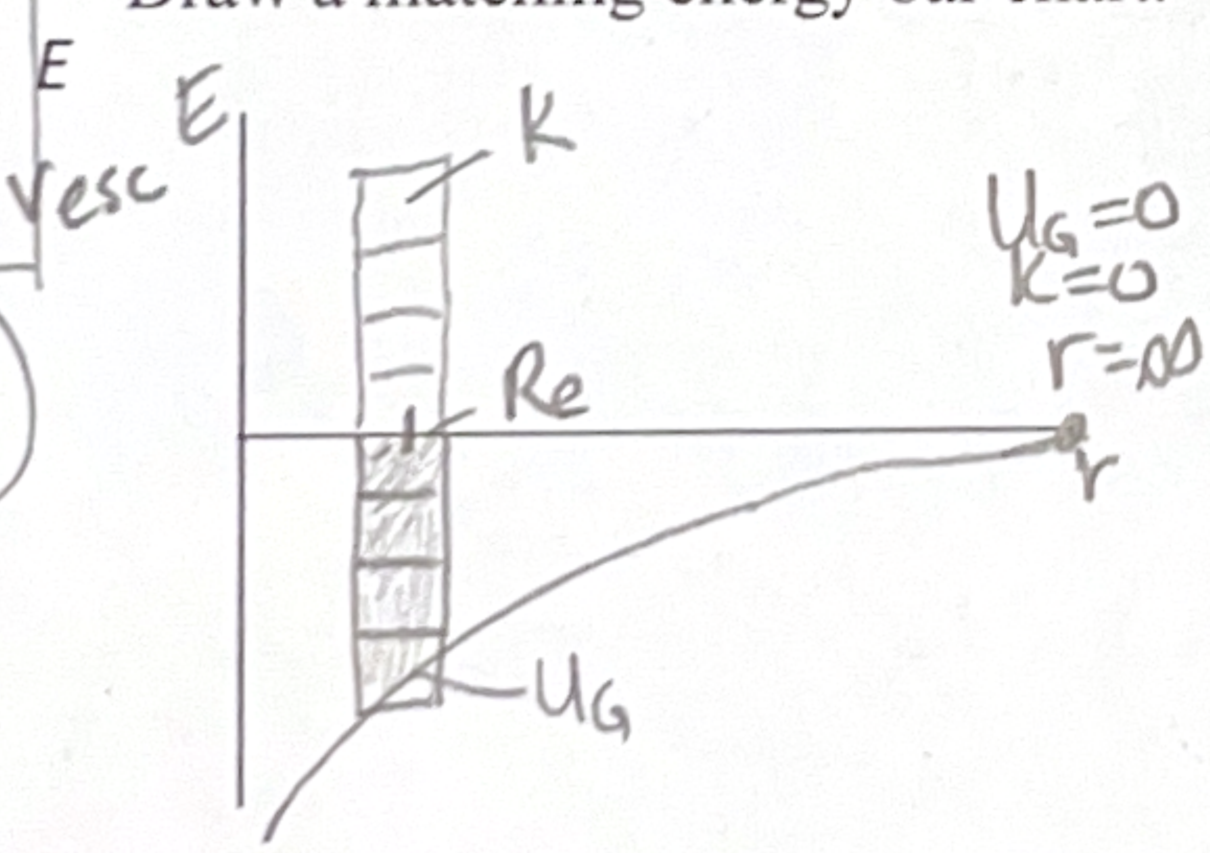


• going faster
• less U_G

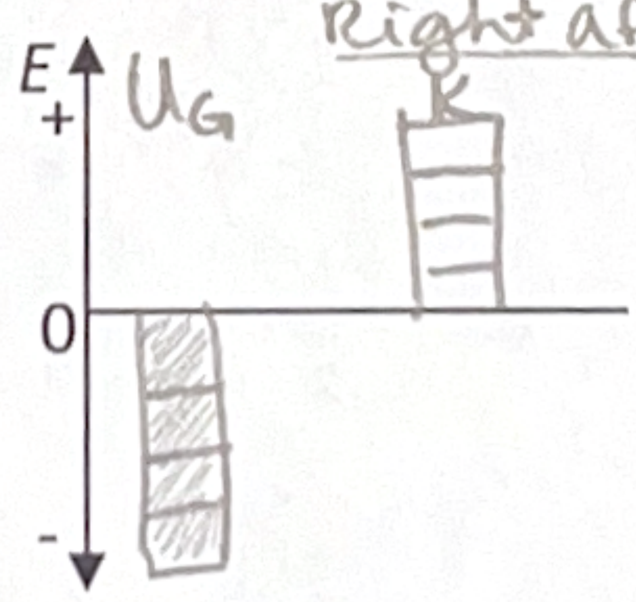
3. A rocket (m_r) is launched from Earth (m_e, R_e) at just the right velocity v_{esc} for it to "escape" the Earth's gravitational field.



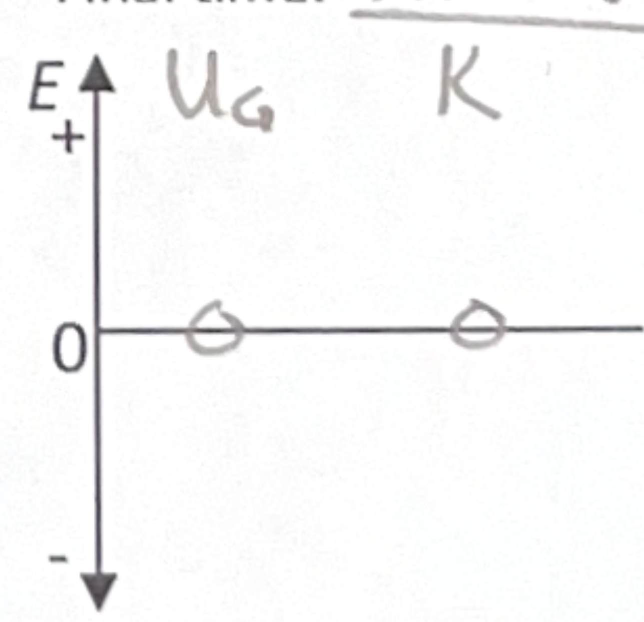
a. Draw a qualitative energy graph showing this process, labeling the initial and final positions. Draw a matching energy bar chart.



Initial time: at Earth's surface Right after launch (A)



Final time: At r=∞ (B)



b. Derive an expression for the escape velocity of the rocket in terms of given variables and fundamental constants.

$E_i + E_{transfers} = E_f$

$U_{GA} + K_A + 0 = U_{GB} + K_B$

$-\frac{Gm_em_r}{R_e} + \frac{1}{2}m_rv_{esc}^2 = 0$

$\frac{1}{2}m_rv_{esc}^2 = \frac{Gm_em_r}{R_e}$

$\boxed{v_{esc} = \sqrt{\frac{2Gm_e}{R_e}}}$