Solutions posted

Name: ____________

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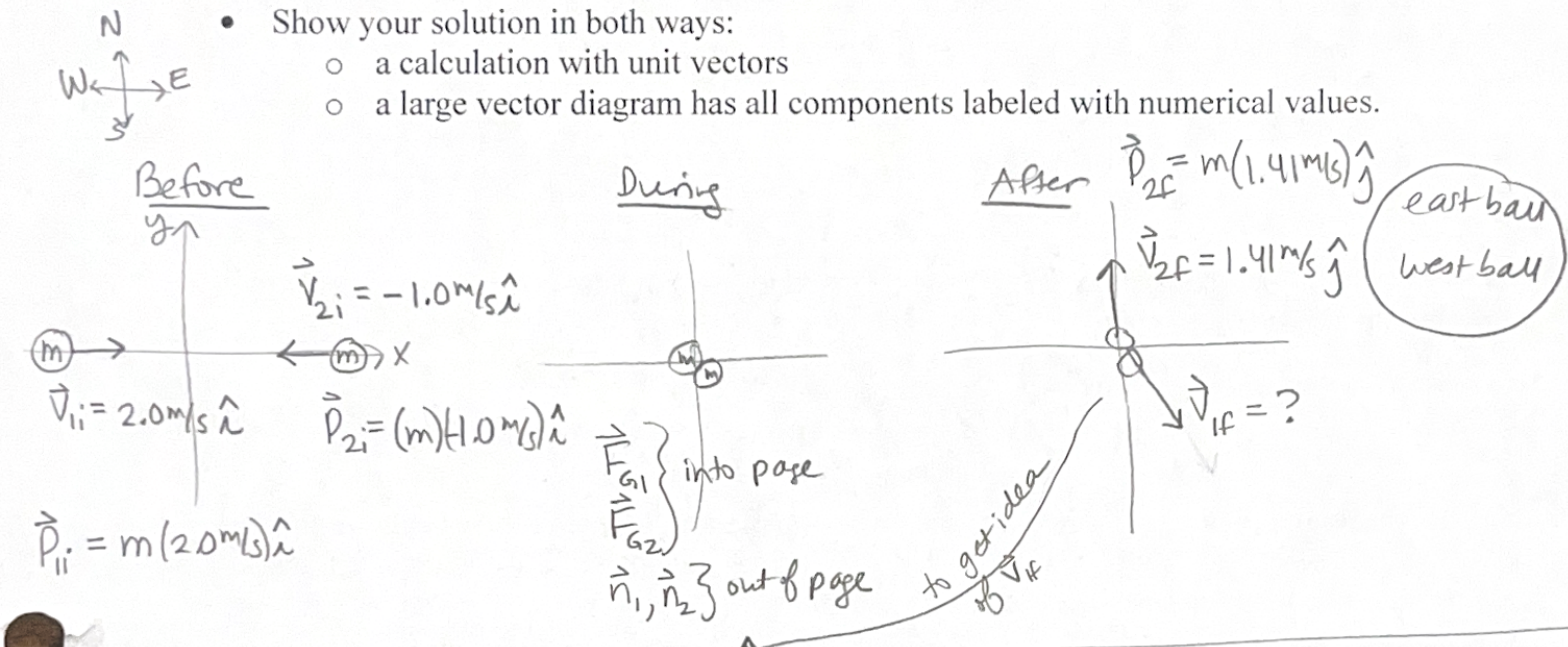
Momentum Application 3

p.289 #48 Glancing collision of billiard balls

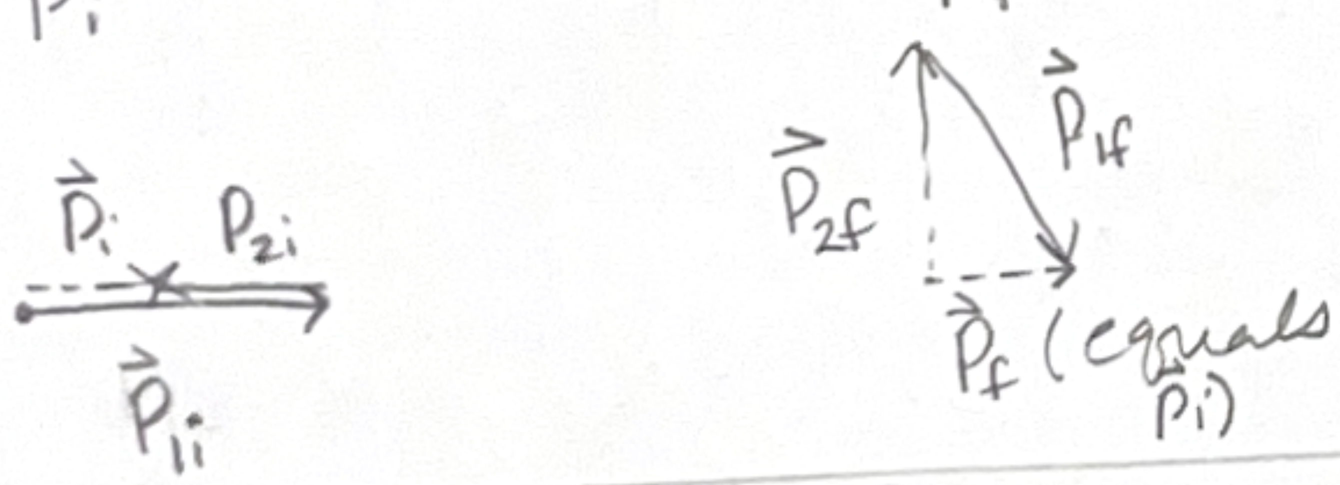
a. Solve the problem as stated. Clearly communicate your thought process. Include:

- *Before, during, after* sketches, coordinate system, system circle
- External force & impulse analysis
- Show your solution in both ways:
  - a calculation with unit vectors
  - a large vector diagram has all components labeled with numerical values.



(initial vector sketch to figure out direction after)

$\vec{P}_i + \Sigma\vec{J} = \vec{P}_f$



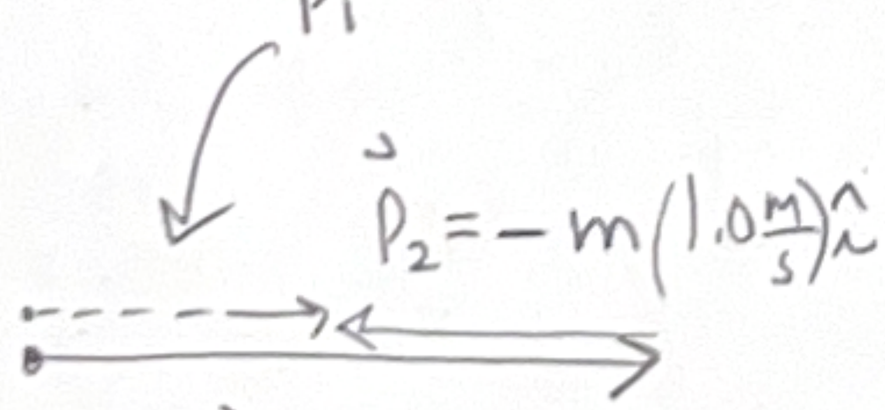
Ext force + Impulse Analysis

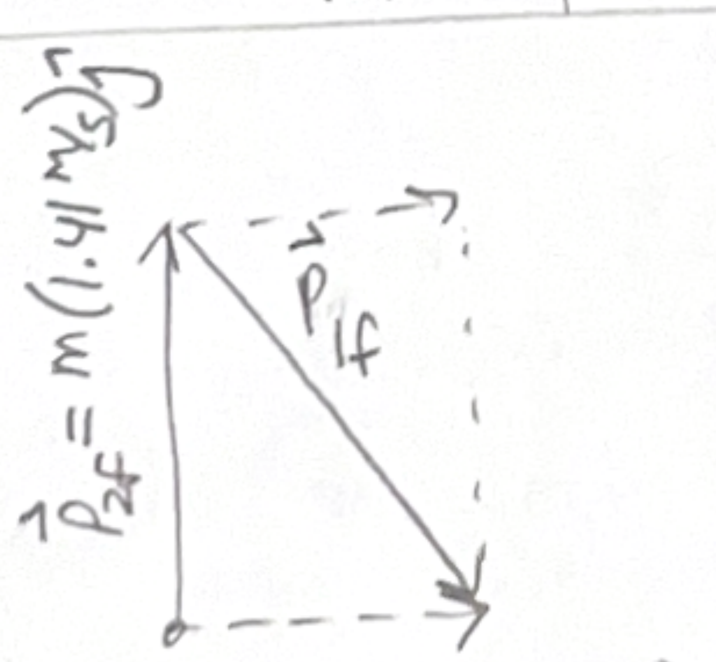
$\vec{F}_{G1} \Rightarrow \vec{J}_1$
$\vec{F}_{G2} \Rightarrow \vec{J}_2$ } cancel
$\vec{n}_1 \Rightarrow \vec{J}_3$
$\vec{n}_2 \Rightarrow \vec{J}_4$ } cancel

$\Sigma\vec{J} = 0$

(Vector Diagram)

System $\vec{P}_i = m(1.0\,m/s)\hat{i}$





$\vec{P}_f = m(1.0\,m/s)\hat{i}$ because it equals $\vec{P}_i$ since $\Sigma\vec{J} = 0$

$\vec{P}_f = \vec{P}_{2f} + \vec{P}_{1f}$

So, components of $\vec{P}_{1f}$ are:

$\vec{P}_{1f} = m(1.0\,m/s)\hat{i} - m(1.41\,m/s)\hat{j}$

$\boxed{\vec{V} = \vec{P}_{1f}/m = (1.0\,m/s)\hat{i} - (1.41\,m/s)\hat{j}}$

(calculations)

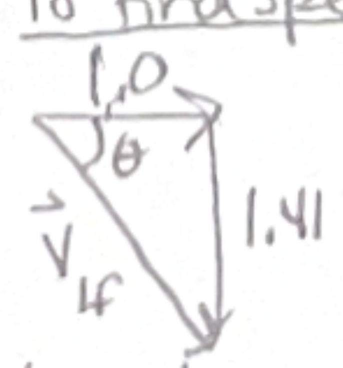
$\vec{P}_i + \Sigma\vec{J} = \vec{P}_f$

$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$

$\cancel{m}(2.0\,m/s)\hat{i} + \cancel{m}(-1.0\,m/s)\hat{i} = \cancel{m}(1.41\,m/s)\hat{j} + \cancel{m}\vec{V}_{1f}$

$\boxed{(1.0\,m/s)\hat{i} - 1.41\,m/s\,\hat{j} = \vec{V}_{1f}}$

To find speed, use pythagorean theorem:



$V_{1f}^2 = (1.0\,m/s)^2 + (1.41\,m/s)^2$

$V_{1f} = 1.73\,m/s$

Use tangent to find θ:

$\theta = \tan^{-1}\left(\frac{1.41}{1.0}\right) = 55°$

$\boxed{\vec{V}_f = (1.73\,m/s,\ 55°\text{ South of East})}$

b. Did each billiard ball experience the same magnitude of force from the other ball during the collision? Would your answer be different if one ball had double the mass of the other? Explain.

Yes, Because they are a Newton's 3rd Law interaction force pair, so they are equal in magnitude and opposite in direction.

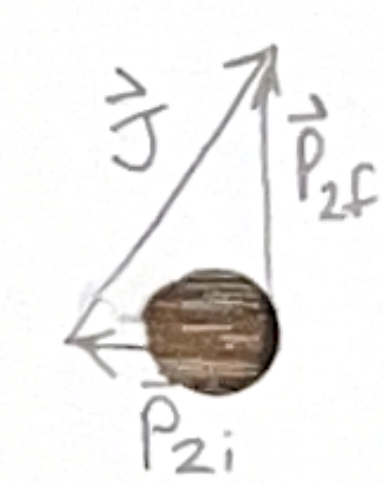
The mass does not matter because even if one is greater mass, they are still an interaction force pair.

c. Did both billiard balls experience a force from the other ball for the same interval of time? Explain. Yes, they are colliding with each other, so once one separates from two, two must have separated from one!

d. Did both billiard balls experience the same magnitude of impulse exerted by the other ball during the collision? Explain.

Yes, same magnitude of impulse because impulse $= \int_{t_i}^{t_f} F\,dt$, and the F and the t_i and t_f are the same for both balls.

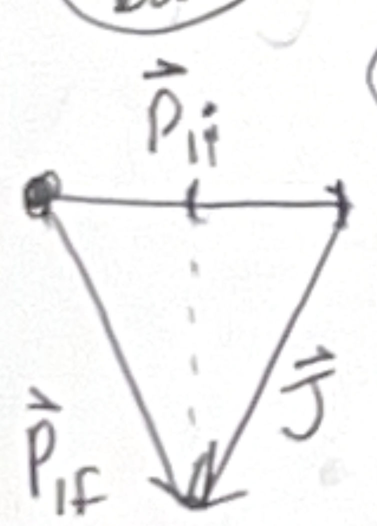
e. Use vector diagrams to figure out the impulse experienced by each billiard ball during the collision, labeling the components of the impulse on each diagram. How do these impulses compare in magnitude and direction?



Choose Ball 1 as the system: Initial momentum of east ball plus impulses on east ball equal final momentum:

east ball

$\vec{P}_{1i} + \vec{J}_{2\,on\,1} = \vec{P}_{1f}$



$\vec{P}_{1i} = m(2.0\,m/s)\hat{i} + \vec{J}_{2\,on\,1} =$ $m(1.0\,m/s)\hat{i}$, $m(-1.41\,m/s)\hat{j}$, $\vec{P}_f$

So, $\vec{J}_{2\,on\,1} = -m(1.0\,m/s)\hat{i} - m(1.41\,m/s)\hat{j}$

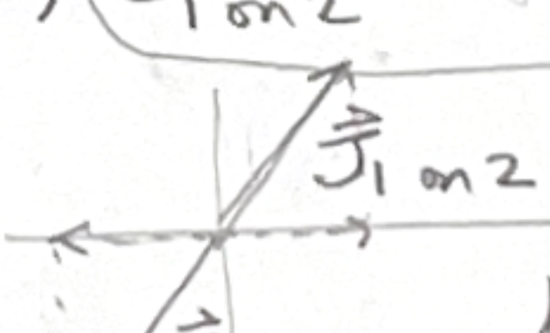
For Ball 2

west ball

$\vec{P}_{2i} + \vec{J}_{1\,on\,2} = \vec{P}_{2f}$

$-m(1.0\,m/s)\hat{i} + \vec{J}_{1\,on\,2} = m(1.41\,m/s)\hat{j}$

So, $\vec{J}_{1\,on\,2} = +m(1.0\,m/s)\hat{j} + m(1.41\,m/s)\hat{j}$



They have same magnitude components but opposite directions, so the impulses are equal in mag. but opp. in direction.

f. Calculate the kinetic energy of the system before the collision, the kinetic energy of the system after the collision, and the change in kinetic energy of the system as a result of the collision. Did the kinetic energy of the system increase, decrease, or stay constant?

$K_i = K_{1i} + K_{2i} = \frac{1}{2}m(2.0\,m/s)^2 + \frac{1}{2}m(1.0\,m/s)^2 = \boxed{2.5\,m^2}$

$K_f = K_{1f} + K_{2f} = \frac{1}{2}m(1.73\,m/s)^2 + \frac{1}{2}m(1.41\,m/s)^2 = \boxed{2.5\,m^2}$

$\boxed{\Delta K = 0}$

Kinetic energy stayed constant