(11)

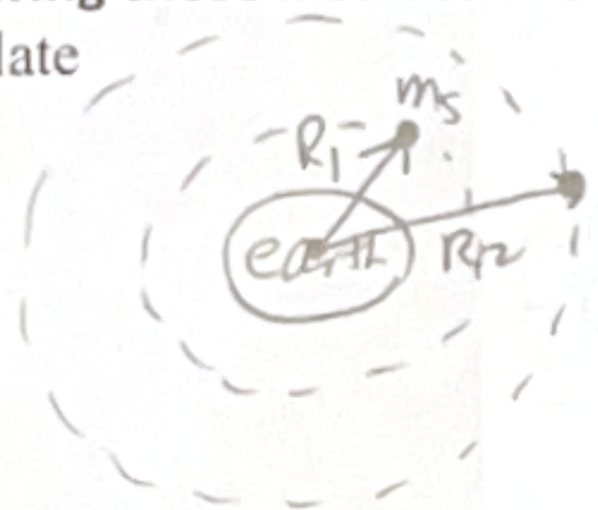
Solutions posted

Name: ____________
Date: ____________

Momentum Application 2

1. p.355 #52, following these instructions:

a. Sketch and translate



$R_e = 6.37\times10^6 m$

$R_1 = 250 km + R_e = 6.62\times10^6 m$

$R_2 = 610 km + R_e = 6.98\times10^6 m$

$m_s = 75000 kg$

$m_e = 5.98\times10^{24} kg$

Energy to change orbits = ?

b. Calculate the speed the satellite must have in each orbit.

circular motion:

$a_r = \frac{\Sigma F_r}{m}$

$\frac{v^2}{R} = \frac{\left(\frac{G m_e m_s}{R^2}\right)}{m_s}$

$V = \sqrt{\frac{G m_e}{R}}$

orbit 1: $V_1 = \sqrt{\frac{G m_e}{R_1}} = \sqrt{\frac{(6.67\times10^{-11} \frac{Nm^2}{kg^2})(5.98\times10^{24} kg)}{6.62\times10^6 m}} = 7762 \text{ m/s}$

orbit 2: $V_2 = \sqrt{\frac{G m_e}{R_2}} = \sqrt{\frac{(6.67\times10^{-11} \frac{Nm^2}{kg^2})(5.98\times10^{24} kg)}{6.98\times10^6 m}} = 7559 \text{ m/s}$

Slower in the higher orbit

c. Calculate the change in potential energy of the satellite-earth system as the satellite changes orbits. By how much does it increase or decrease?

$U_{G1} = -\frac{G m_e m_s}{R_1}$

$U_{G2} = -\frac{G m_e m_s}{R_2}$

$\Delta U_G = U_{G2} - U_{G1}$

$= -\frac{G m_e m_s}{R_2} - \left(-\frac{G m_e m_s}{R_1}\right)$

$= -G m_e m_s \left(\frac{1}{R_2} - \frac{1}{R_1}\right)$

$= -(6.67\times10^{-11} \frac{Nm^2}{kg^2})(5.98\times10^{24} kg)(75000 kg)\left(\frac{1}{6.98\times10^6 m} - \frac{1}{6.62\times10^6 m}\right)$

$= -2.99\times10^{19}\left(\frac{1}{(\ )} - \frac{1}{(\ )}\right)$

$= \boxed{2.33\times10^{11} J}$

increased because it is + and they are farther apart

d. Calculate the change in kinetic energy of the satellite-earth system as the satellite changes orbits. By how much does it increase or decrease?

$K_1 = \frac{1}{2} m_s V_1^2 = \frac{1}{2}(75000 kg)(7762 \text{ m/s})^2 = 2.26\times10^{12} J$

$K_2 = \frac{1}{2} m_s V_2^2 = \frac{1}{2}(75000 kg)(7559 \text{ m/s})^2 = 2.14\times10^{12} J$

$\Delta K = K_2 - K_1 = 2.14\times10^{12} J - 2.26\times10^{12} J = \boxed{-1.2\times10^{11} J}$ It decreased by $1.2\times10^{11} J$

e. Calculate the total energy of the system for the first orbit and the total energy of the system for the second orbit. How much energy was required to boost the satellite to the new orbit?

① $TE_1 = U_{G1} + K_1$

$= -\frac{G m_e m_s}{R_1} + K_1$

$= -\frac{(6.67\times10^{-11} \frac{Nm^2}{kg^2})(5.98\times10^{24} kg)(75000 kg)}{6.62\times10^6 m} + 2.26\times10^{12} J$

$= -2.26\times10^{12} J$

② Do same calc for TE_2, and

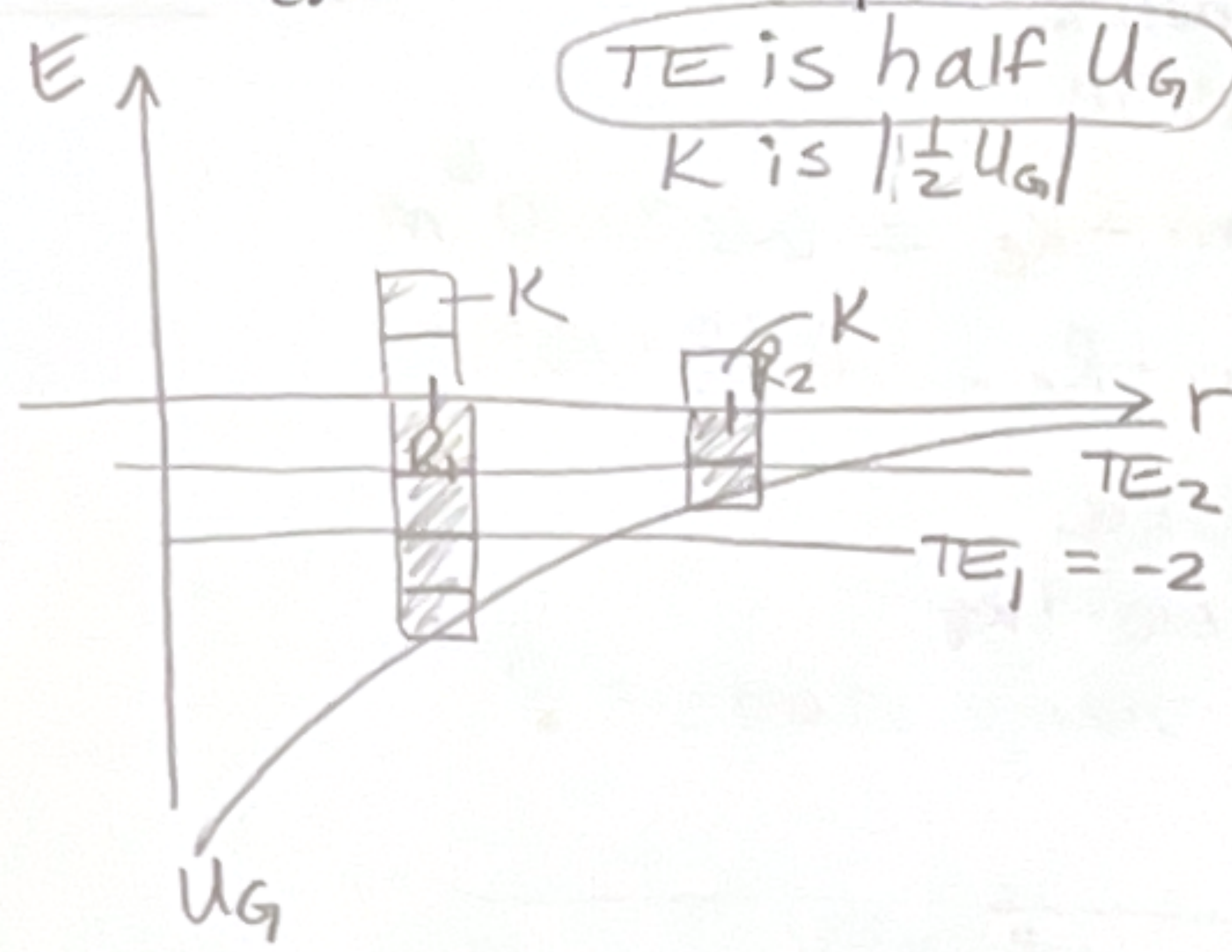
$TE_2 = -2.14\times10^{12} J$

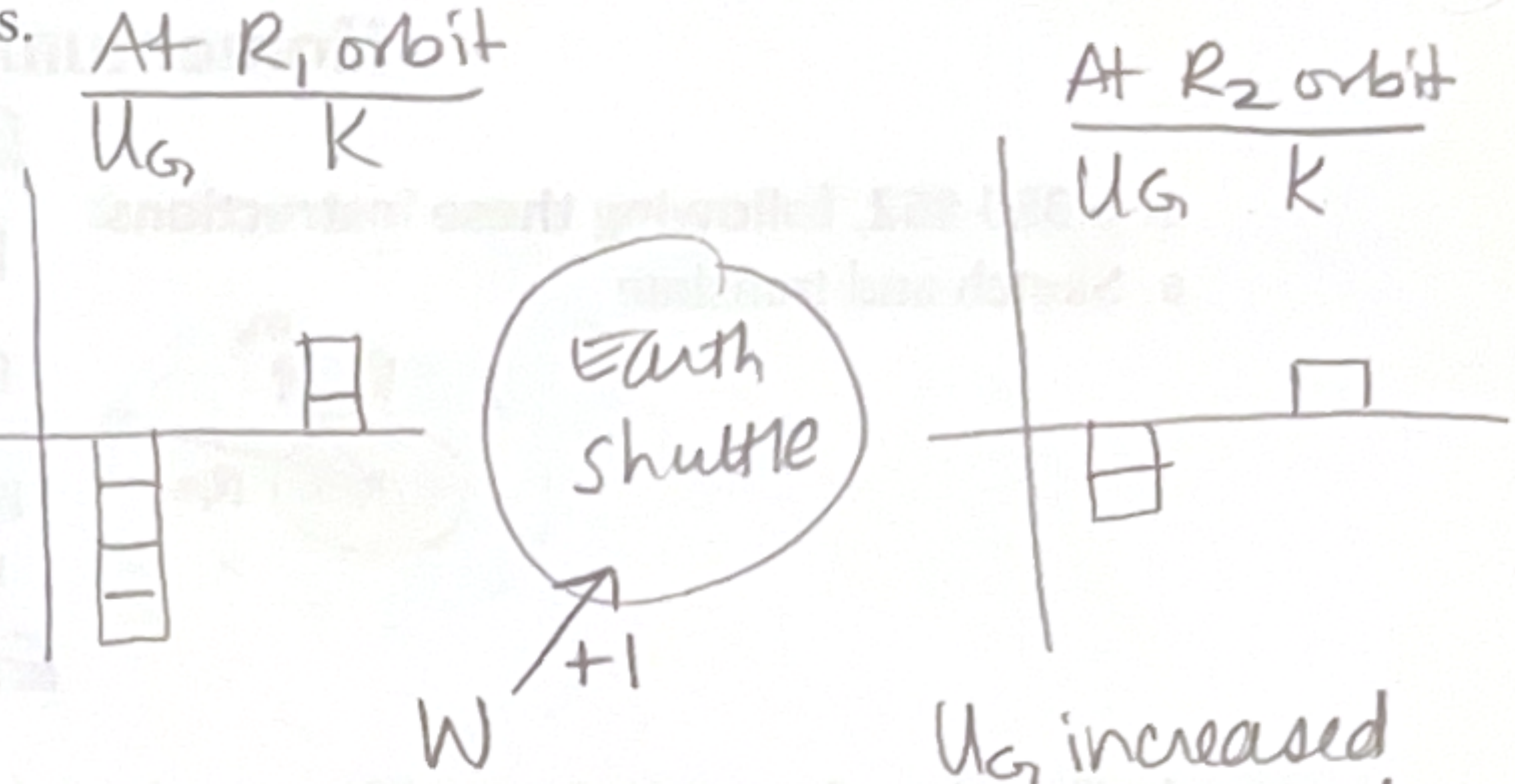
③ $\Delta E = TE_2 - TE_1$

$\Delta E = -2.14\times10^{12} J - (-2.26\times10^{12} J)$

$\Delta E = 1.14\times10^{11} J$

f. Draw a qualitative energy graph showing UG, K and TE showing both orbits, and a matching energy bar chart for the process of changing orbits.

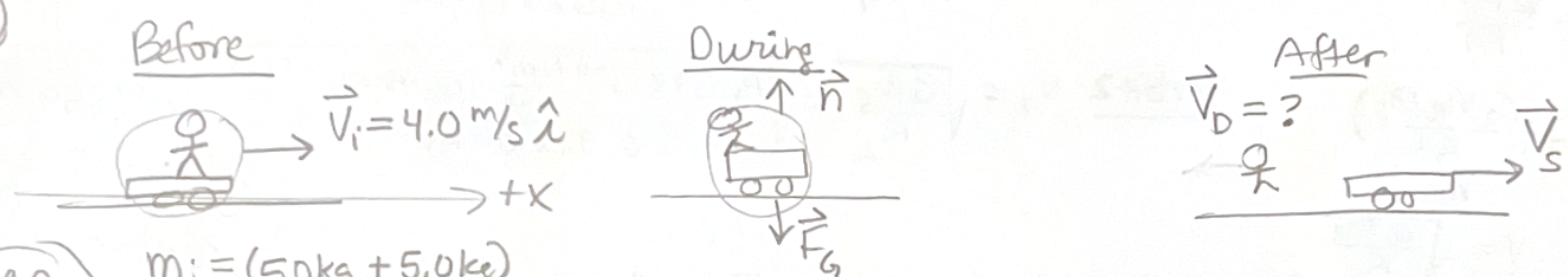




U_G increased
K decreased
TE increased

2. p.288 #28 Show the problem-solving steps

(1)



Dan
Skateboard

$m_i = (50\,kg + 5.0\,kg)$
$= 55\,kg$

(His feet pushing on the skateboard is an internal force)

(2)

Ext force + Impulse

$\vec{n} \Rightarrow \vec{J}_n$ cancel
$F_G \Rightarrow \vec{J}_{F_G}$

$\therefore\ \Sigma\vec{J} = 0$

momentum conservation
vector diagram

$\vec{p}_i + \Sigma\vec{J} = \vec{p}_f$

$\xrightarrow{\vec{p}_i} + 0 = \xrightarrow{\vec{p}_f}$

This will be $\vec{p}_S + \vec{p}_D$

Before	$\vec{p}_i = m_i\vec{V}_i = 55\,kg\,(4.0\,m/s\,\hat{i}) = 220\,\frac{kg\,m}{s}\,\hat{i}$
During	$\Sigma\vec{J} = 0$
After	Dan: $\vec{p}_D = m_D\vec{V}_D = (50\,kg)(\vec{V}_D)$
After	Skateboard: $\vec{p}_S = m\vec{V}_S = (5\,kg)(8.0\,m/s\,\hat{i}) = 40\,\frac{kg\,m}{s}\,\hat{i}$

(3) Find $\vec{V}_D$: $\vec{p}_i + \Sigma\vec{J} = \vec{p}_f$

Dan's momentum
skateboard's momentum

$220\,\frac{kg\,m}{s}\,\hat{i} + 0 = (50\,kg\,\vec{V}_D) + (40\,\frac{kg\,m}{s}\,\hat{i})$

$220\,\frac{kg\,m}{s}\,\hat{i} - 40\,\frac{kg\,m}{s}\,\hat{i} = 50\,kg\,\vec{V}_D$

$\frac{180\,\frac{kg\,m}{s}\,\hat{i}}{50\,kg} = \vec{V}_D$

$\boxed{3.6\,m/s\,\hat{i} = \vec{V}_D}$

He is still moving forward!

He is moving at 3.6 m/s

(4) verify

$\vec{p}_i + \Sigma\vec{J} = \vec{p}_f$

$\vec{p}_i = 220\,\frac{kg\,m}{s}$

$\vec{p}_f = \vec{p}_D + \vec{p}_S$

$\vec{p}_D$ $\vec{p}_S$
$180\,kg\,m/s$ $140\,\frac{kg\,m}{s}$

$180 + 40$
$= 220\,\frac{kg\,m}{s}$

Both sides are equal