Solutions posted

Name: ___________

Date: ___________

Launching Rockets

Instructions: Show your thought process.

1. **New Facts**: Write these into your booklet and memorize.

Energy

- *Mechanical energy* is the sum of a system's kinetic and potential energies
- The potential energy function associated with the gravitational force $F_G = \frac{Gm_1m_2}{r^2}$ is $U_G = -\frac{Gm_1m_2}{r}$.
- *Escape speed* is the launch speed needed for an object to slow to a stop as it reaches infinity.
- The negative sign for energy means "less than 0 joules by"
- On a graph of energy vs. position, higher on the graph means more energy.

Momentum

- Momentum is $\vec{p} = m\vec{v}$
- The law of conservation of momentum is:

$$\Delta\vec{p} = \Sigma\,\overrightarrow{Impulses} \quad \text{which is the same as} \quad \vec{p}_i + \Sigma\,\overrightarrow{Impulses} = \vec{p}_f$$

- The $\overrightarrow{Impulse}$ imparted by a single force that acts from t_i to t_f is $\int_{t_i}^{t_f} \vec{F}dt$

2. **Launching rockets**

A rocket (mass m_r) is launched straight up from large object A's surface at an initial speed of v_{esc}, which is the launch speed needed for the rocket to "escape" from the gravitational pull of the object A and never return. Assume that object A is not rotating. (Object A has mass M_A and radius R_A.)

a. Derive an expression for the escape speed v_{esc} in terms of M_A, R_A, and fundamental constants. Include an energy bar chart with your solution.

Include a sketch, energy graph (showing UG and K at the initial and final positions, and the TE line), and matching energy bar chart with your solution.

b. Calculate the escape speed if (i) the rocket was launched from Earth's surface, and (ii) the rocket was launched from the surface of Mars.

c. If the rocket was launched from Earth's surface with an initial speed that is 2000 m/s more than the escape speed, calculate how fast it is moving when it escapes.

Include a sketch, energy graph (showing UG and K at the initial and final positions, and the TE line), and matching energy bar chart with your solution.

d. What will happen to the rocket if it is launched from Earth's surface with an initial speed that is less than the escape speed?

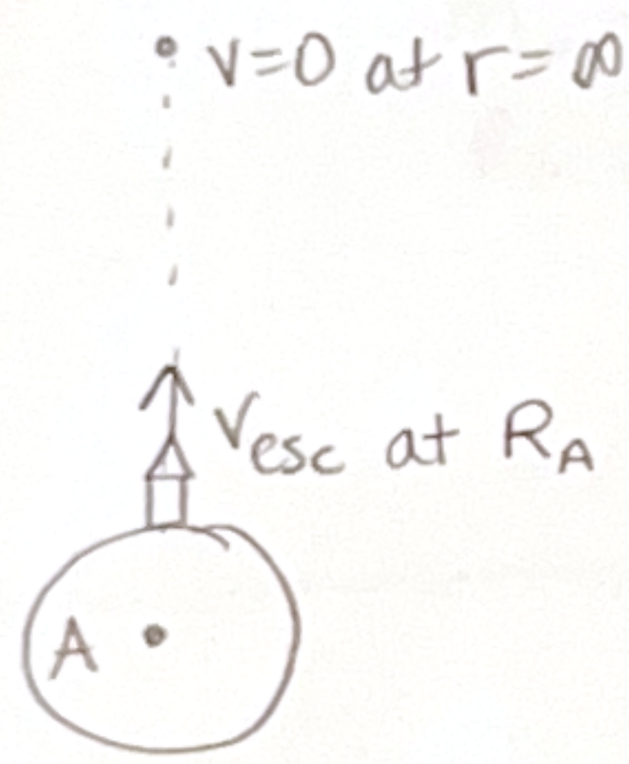
e. If the rocket was launched from Earth's surface with an initial speed of 10,000 km/h, calculate the maximum height it reaches above Earth's surface.

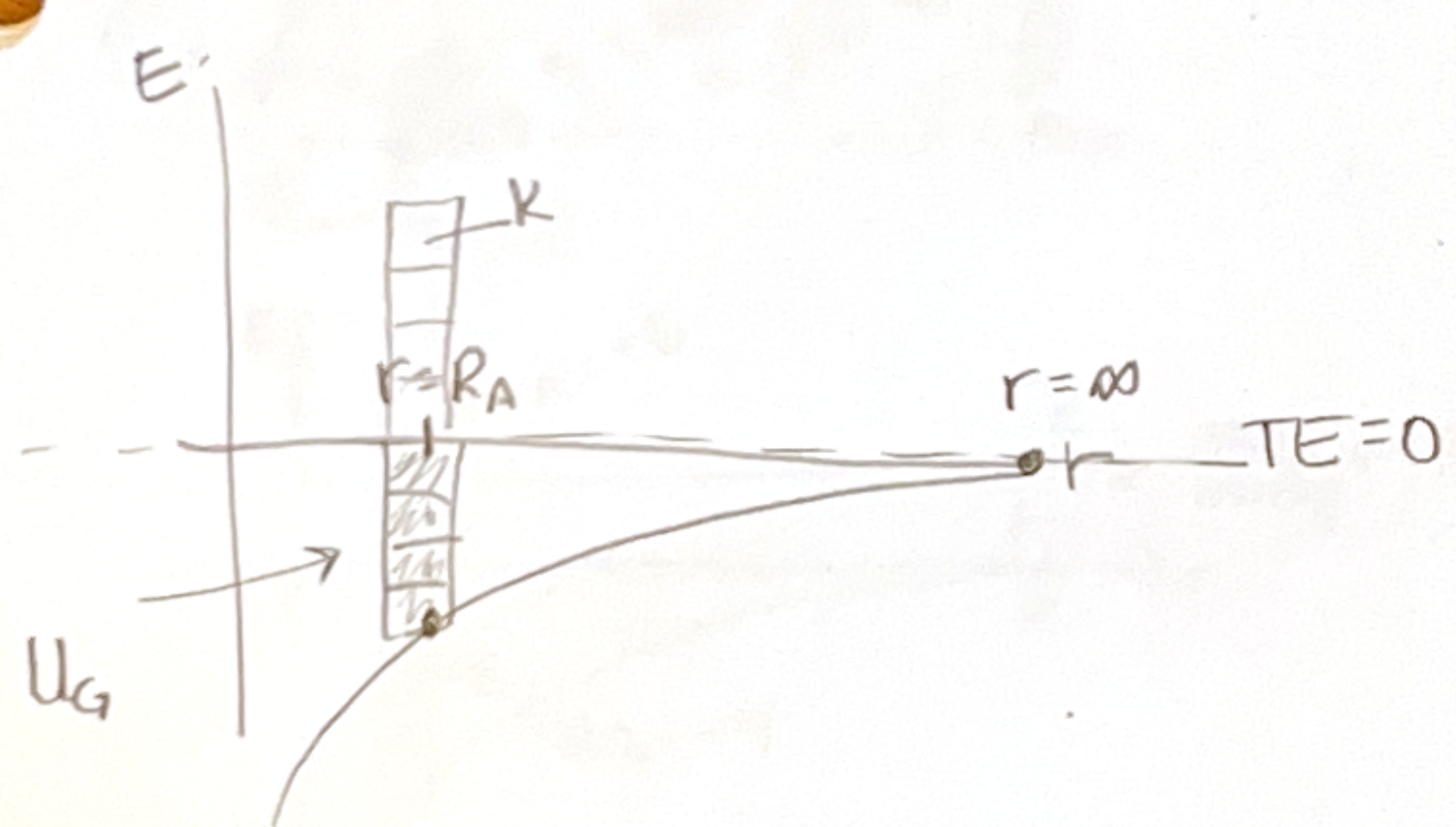
Include a sketch, energy graph (showing UG and K at the initial and final positions, and the TE line), and matching energy bar chart with your solution.

2. (There is no #2!)

③

ⓐ





<u>mathematical representation</u>

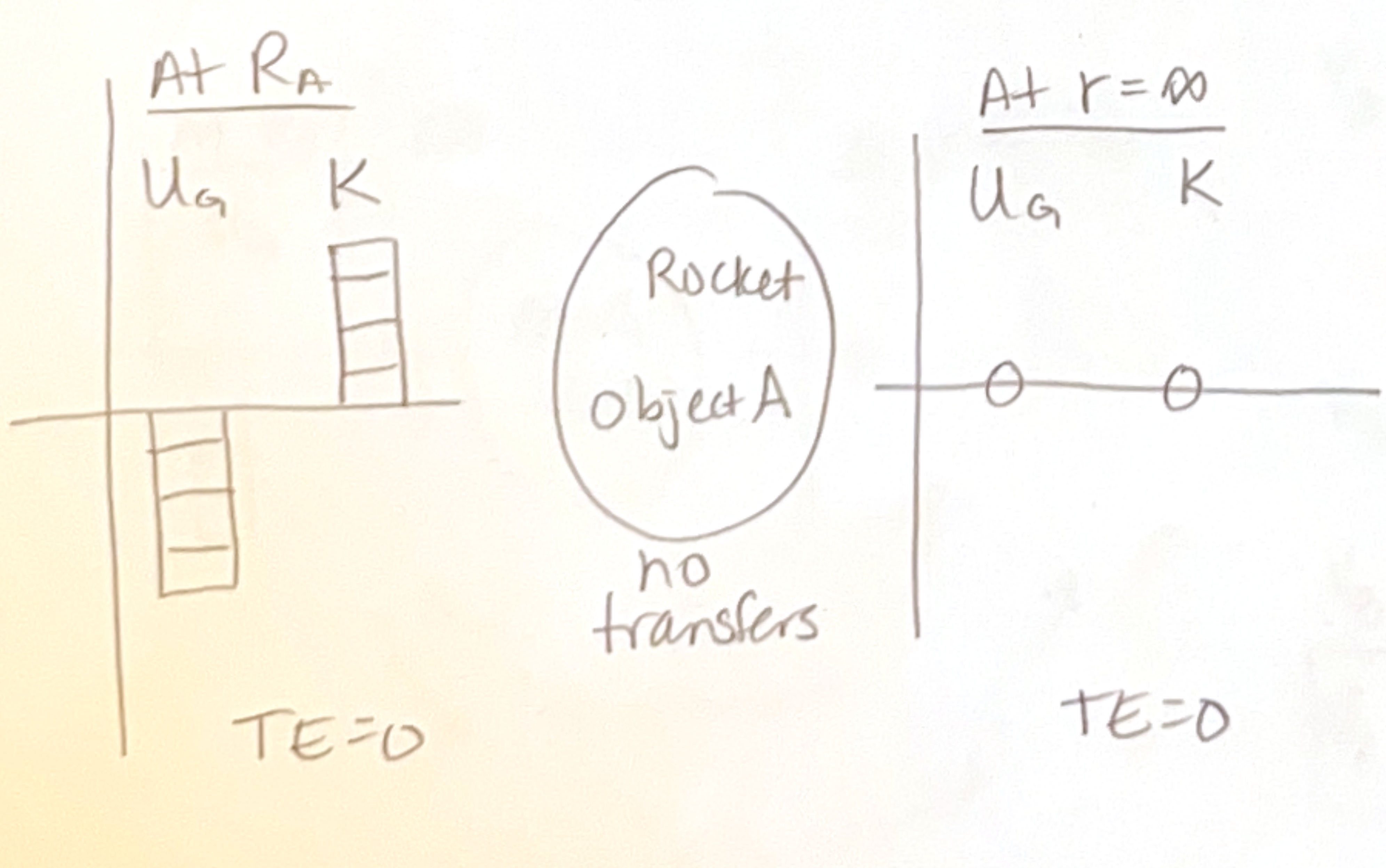
$$E_i + \Sigma \text{transfers} = E_f$$

$$U_{Gi} + K_i + 0 = U_{Gf} + K_f$$

$$-\frac{G m_A m_R}{R_A} + \frac{1}{2} m_R v_{esc}^2 = 0 + 0$$

$$\frac{1}{2} m_R v_{esc}^2 = \frac{G m_A m_R}{R_A}$$

$$\boxed{v_{esc} = \sqrt{\frac{2 G m_A}{R_A}}}$$



The bar chart & energy graph match because they show the same U_G, K, and TE at both times.

(b) From Earth's surface:

$m_e = 5.98 \times 10^{24}\,kg$

$R_e = 6.37 \times 10^{6}\,m$

$$V_{esc} = \sqrt{\frac{2Gm_e}{R_e}}$$

$$V_{esc} = \sqrt{\frac{2(6.67\times10^{-11}\,Nm^2/kg^2)(5.98\times10^{24}\,kg)}{(6.37\times10^{6}\,m)}}$$

$$V_{esc} = \sqrt{1.25\times10^{8}\,\frac{m\,N}{kg}}$$

$$\frac{m}{kg}\cdot\left(\frac{kg\,m}{s^2}\right) \leftarrow N$$

$$= m^2/s^2$$

$$\boxed{V_{esc} = 11191\ m/s}$$

From surface of Mars:

$m_m = 6.42 \times 10^{23}\,kg$

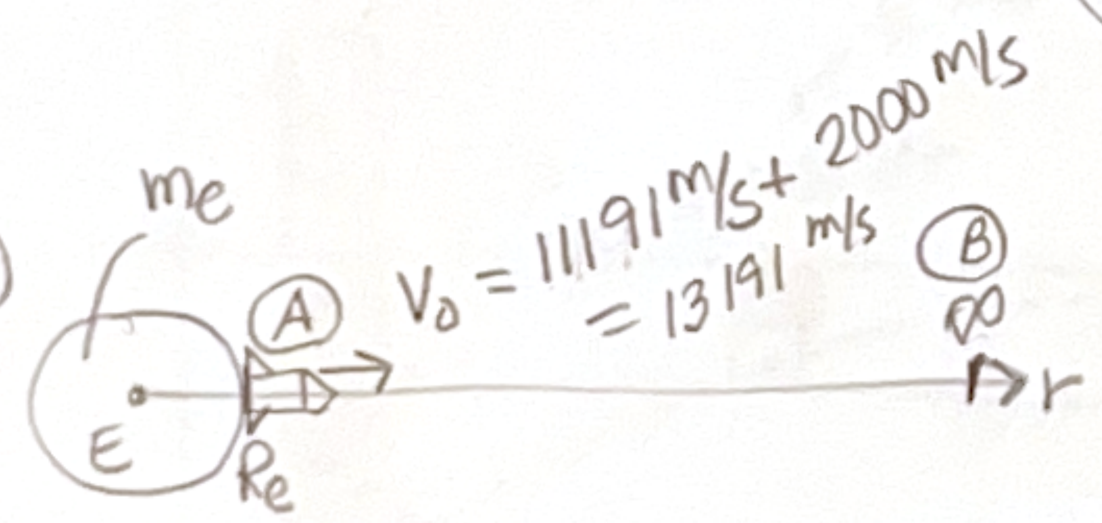
$R_m = 3.37 \times 10^{6}$

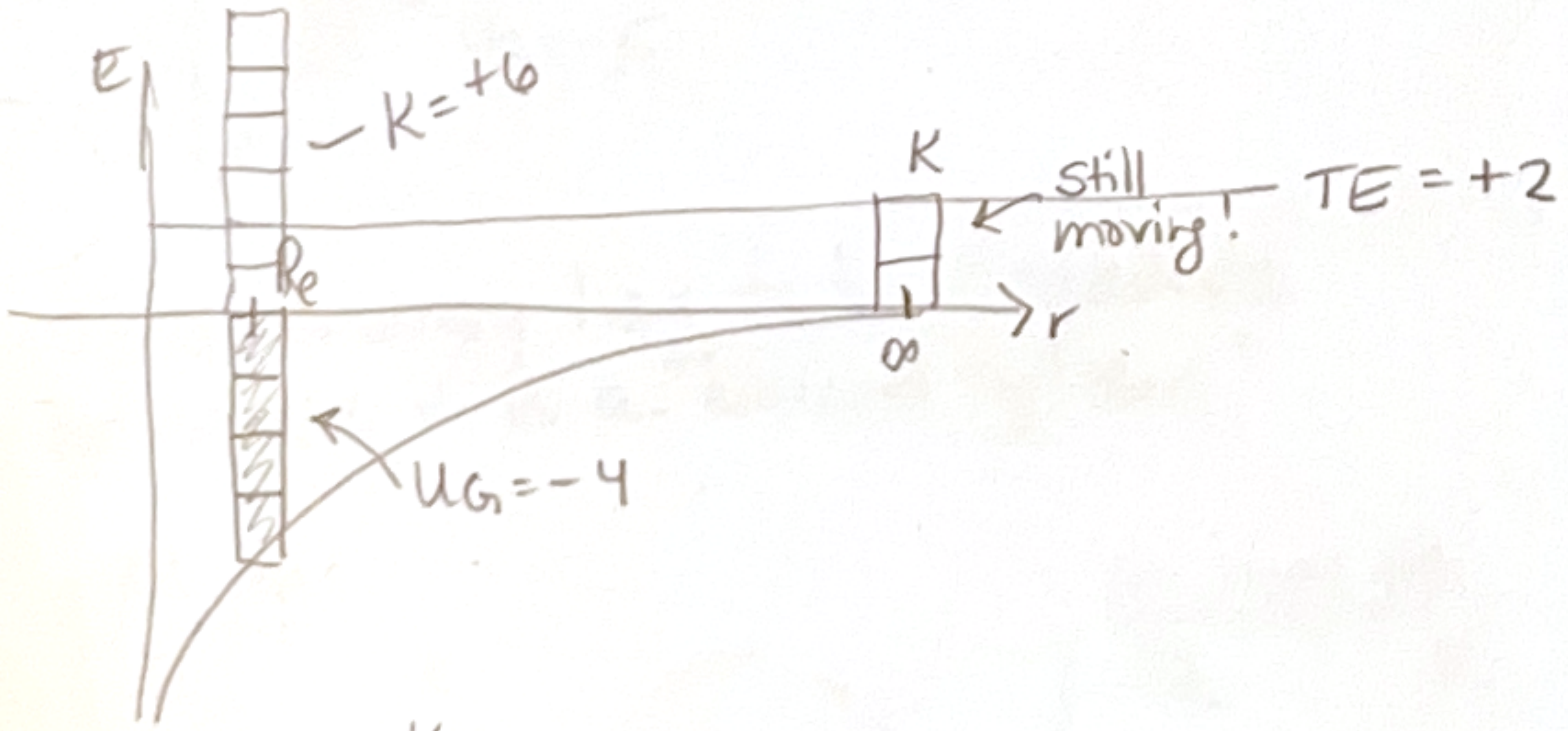
$$V_{esc} = \sqrt{\frac{2Gm_m}{R_m}}$$

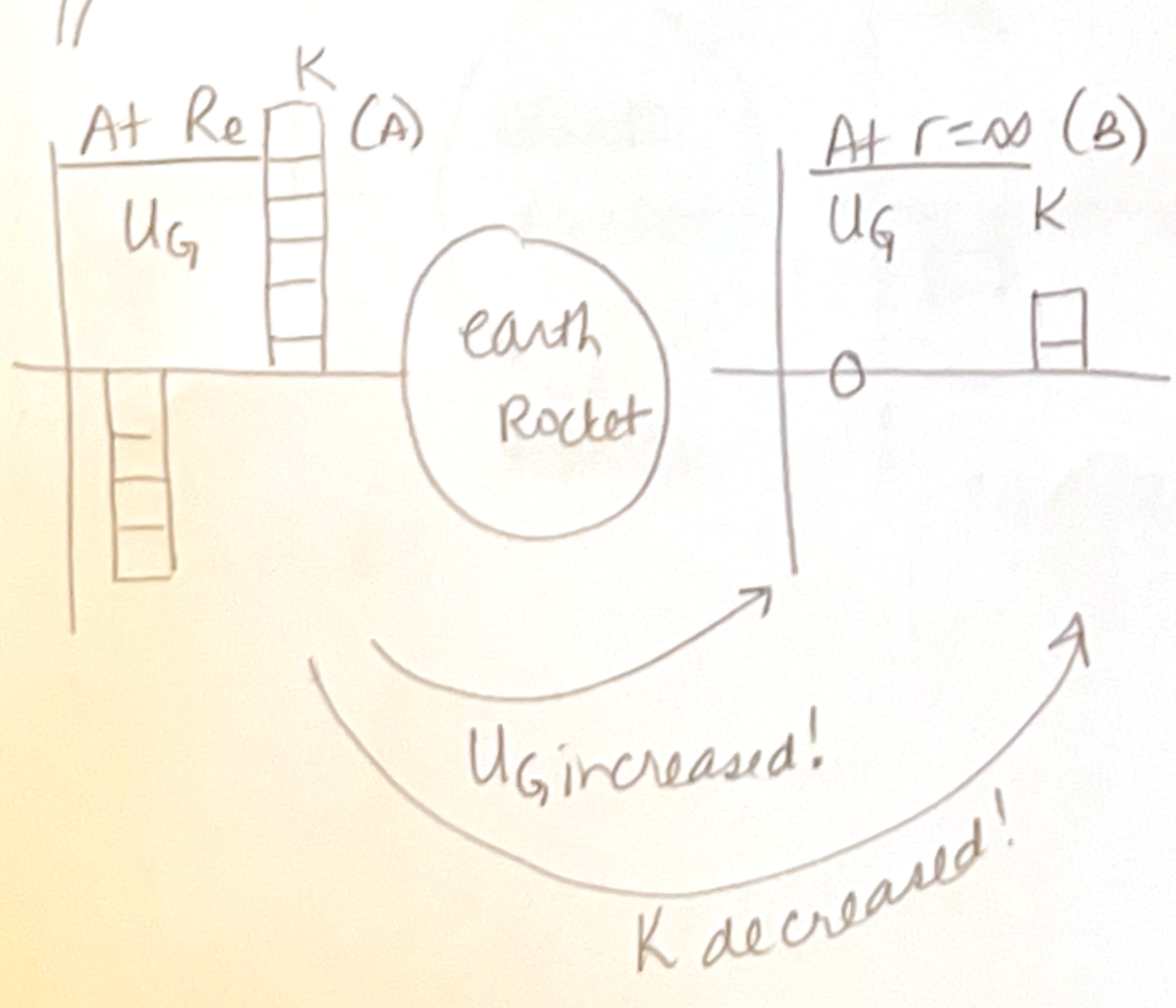
$$V_{esc} = \sqrt{\frac{2(6.67\times10^{-11}\,Nm^2/kg^2)(6.42\times10^{23}\,kg)}{(3.37\times10^{6}\,m)}}$$

$$\boxed{V_{esc} = 5041\ m/s}$$ which is less than from Earth.

(c)







$$E_i + \Sigma transfers = E_f$$

$$U_{GA} + K_A = K_B$$

$$\frac{-Gm_e \cancel{m_R}}{R_e} + \frac{1}{2}\cancel{m_R}V_0^2 = \frac{1}{2}\cancel{m_R}V_B^2$$

$$V_B = \sqrt{2\left(\frac{-Gm_e}{R_e} + \frac{1}{2}V_0^2\right)}$$

$$V_B = \sqrt{2\left[\left(\frac{-6.67\times10^{-11}\frac{Nm^2}{kg^2}(5.98\times10^{24}\,kg)}{6.37\times10^{6}m}\right) + \frac{1}{2}(13191\,m/s)^2\right]}$$

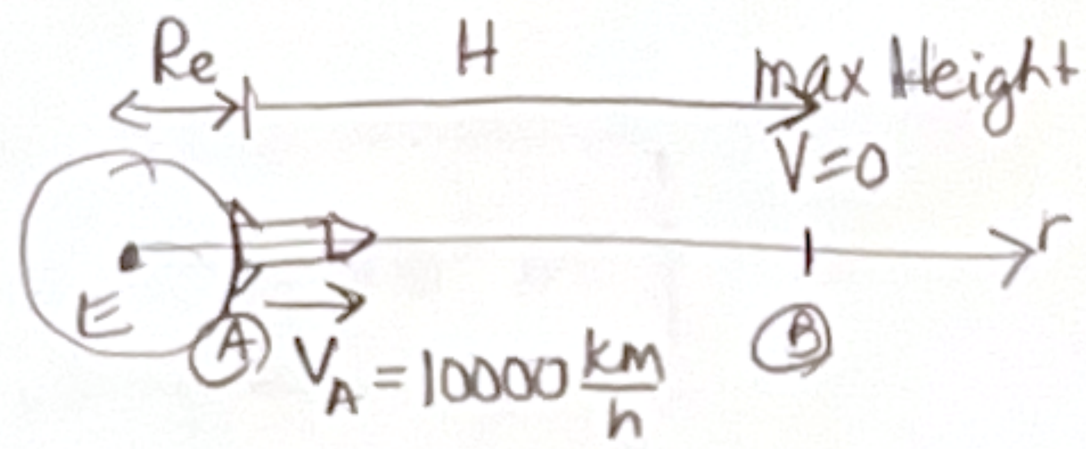
$$V_B = \sqrt{2\left(-62616327\frac{m^2}{s^2} + 87001241\frac{m^2}{s^2}\right)}$$

$$V_B = \sqrt{48769827\frac{m^2}{s^2}}$$

$$\boxed{V_B = 6984\ m/s}$$

(d) If the rocket is launched with $v_0 < v_{esc}$, the rocket will not escape. When it reaches its max height where $v=0$, it will be pulled back toward earth because $\vec{F}_G \neq 0$ at the max height!

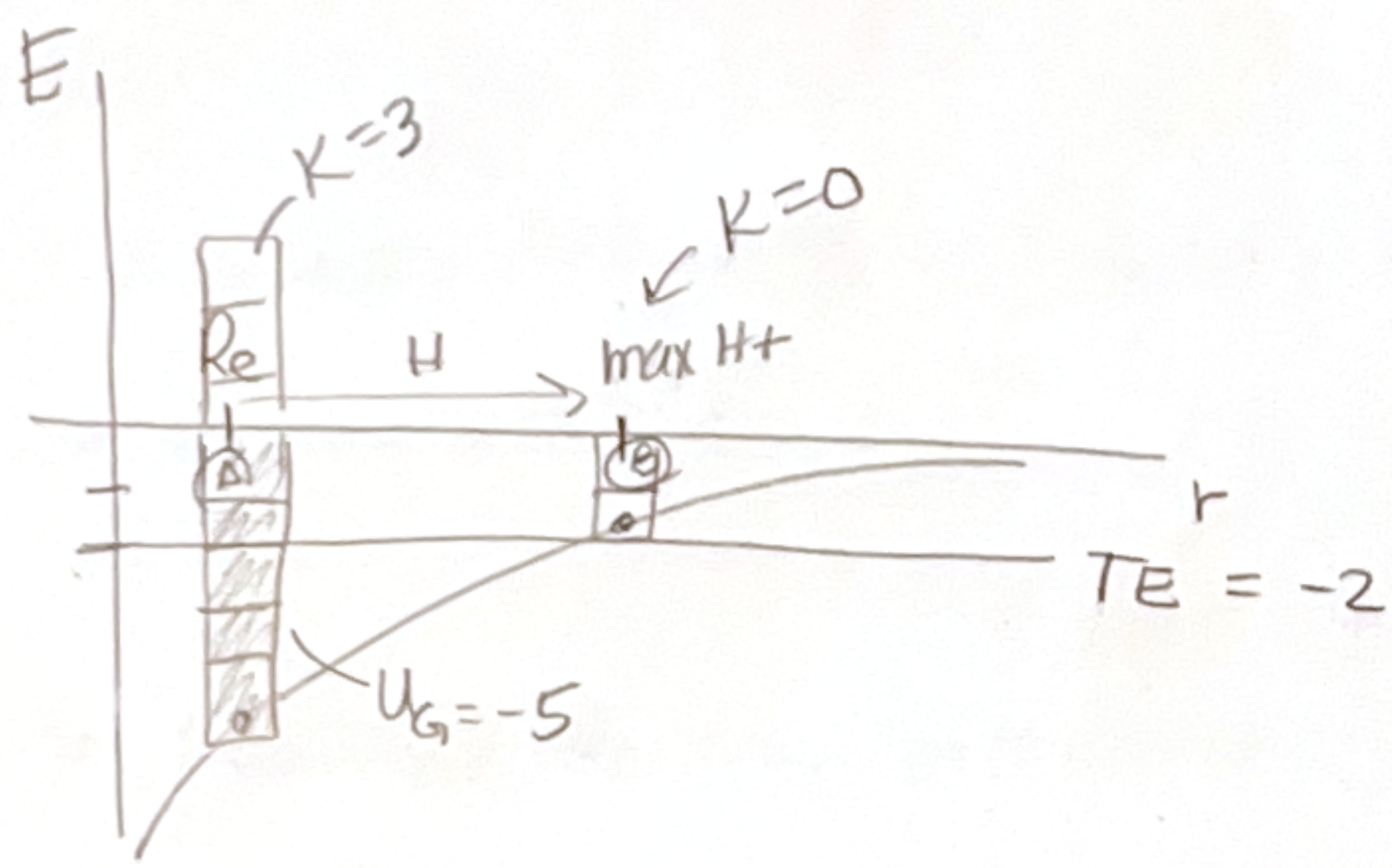
(e)



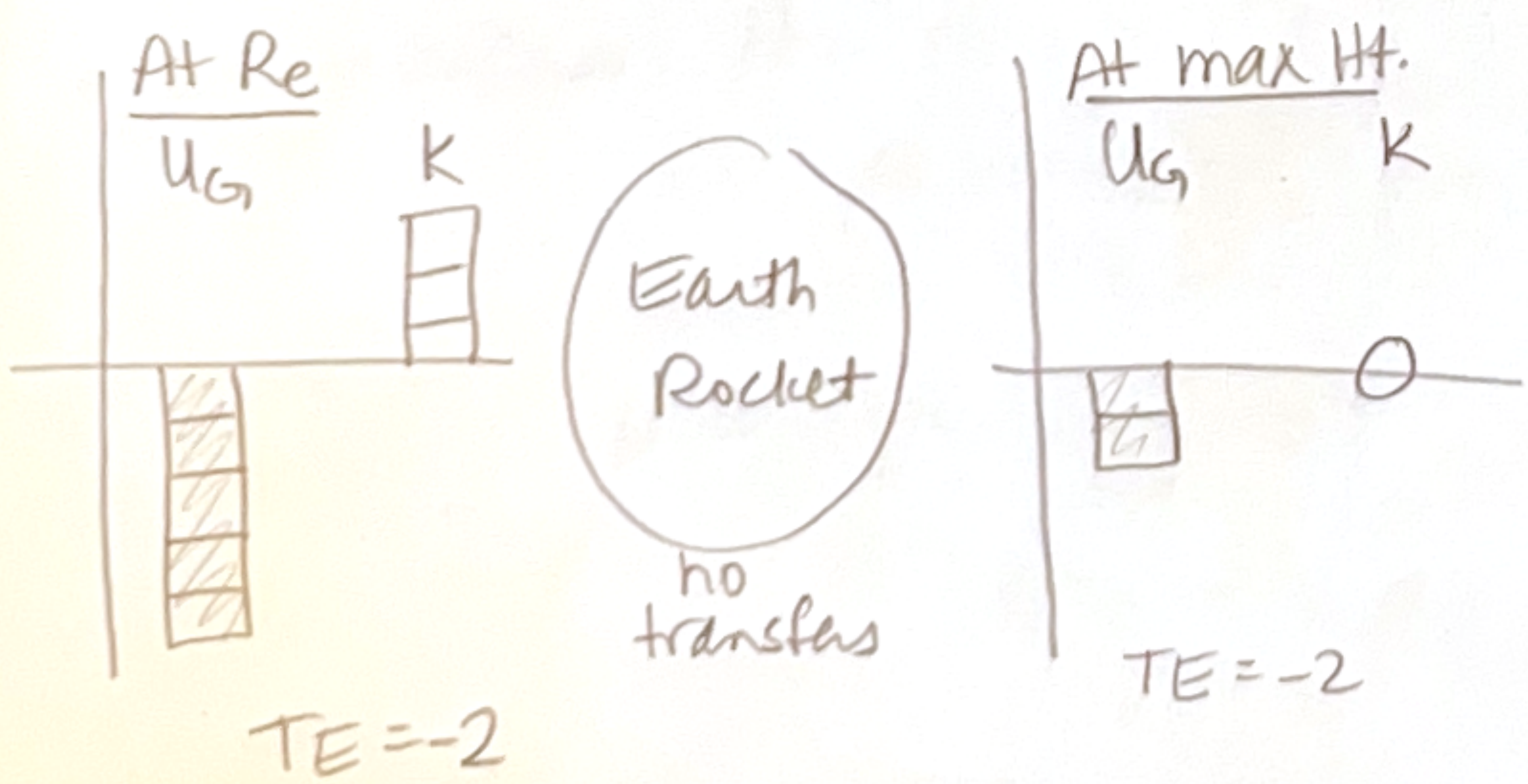
H is measured from Earth's surface.

Convert V_A to m/s: $10000 \frac{km}{h} \left(\frac{1h}{3600s}\right)\left(\frac{1000m}{1km}\right)$

$V_A = 2778$ m/s



Total energy has to be negative because it is a "bound system."



Find H

$$E_i + \Sigma \text{transfers} = E_f$$

$$U_{GA} + K_A = U_{GB}$$

(it is at rest at max Ht.) so $V_B = 0$

$$-\frac{G m_e \cancel{m_R}}{R_e} + \frac{1}{2}\cancel{m_R} V_A^2 = -\frac{G m_e \cancel{m_R}}{(R_e + H)}$$

$$-\frac{G m_e}{R_e} + \frac{1}{2}V_A^2 = -\frac{G m_e}{(R_e+H)}$$

Divide both sides by $-Gm_e$ so it only appears in one place

$$\frac{+1}{R_e} - \frac{V_A^2}{2Gm_e} = \frac{1}{(R_e+H)}$$

multiply both sides by (R_e+H)

$$(R_e+H)\left[\frac{1}{R_e} - \frac{V_A^2}{2Gm_e}\right] = 1$$

Divide both sides by the [ ]

$$R_e + H = \left[\frac{1}{R_e} - \frac{V_A^2}{2Gm_e}\right]^{-1}$$

means $\frac{1}{[\ ]}$

$$\boxed{H = \left[\frac{1}{R_e} - \frac{V_A^2}{2Gm_e}\right]^{-1} - R_e}$$

$$H = \left[\frac{1}{6.37\times10^6 m} - \frac{(2778\,m/s)^2}{2(6.67\times10^{-11}\frac{Nkg^2}{m^2})(5.98\times10^{24}kg)} \right]^{-1} - 6.37\times10^6 m$$

$$H = \left[1.569858713\times10^{-7} - 9.674030878\times10^{-9} \right]^{-1} - 6.37\times10^6 m$$

$$\boxed{H = 4.2\times10^5 m}$$

Doing the algebra differently...

- start here: $-\frac{Gme}{Re} + \frac{1}{2}V_A^2 = \frac{-Gme}{Re+H}$
  - multiply both sides by $^-(Re+H)$: $(Re+H)\left[\frac{Gme}{Re} - \frac{1}{2}V_A^2\right] = Gme$
  - divide both sides by [ ]: $Re+H = \frac{Gme}{[\quad]}$
  - Subtract Re from both sides: $H = \frac{Gme}{\left(\frac{Gme}{Re} - \frac{V_A^2}{2}\right)} - Re$
  - put in values: $H = \frac{(6.67\times10^{-11}\frac{Nm^2}{kg^2})(5.98\times10^{24}kg)}{\left(\frac{(6.67\times10^{-11}\frac{Nm^2}{kg^2})(5.98\times10^{24}kg)}{6.37\times10^6 m} - \frac{(2778\,m/s)^2}{2}\right)} - 6.37\times10^6 m$

$$H = \frac{3.98866\times10^{14}}{(58757684.53)} - 6.37\times10^6 m$$

$$H = 6788326.595\, m - 6.37\times10^6 m$$

$$H = \boxed{4.2\times10^5 m}$$