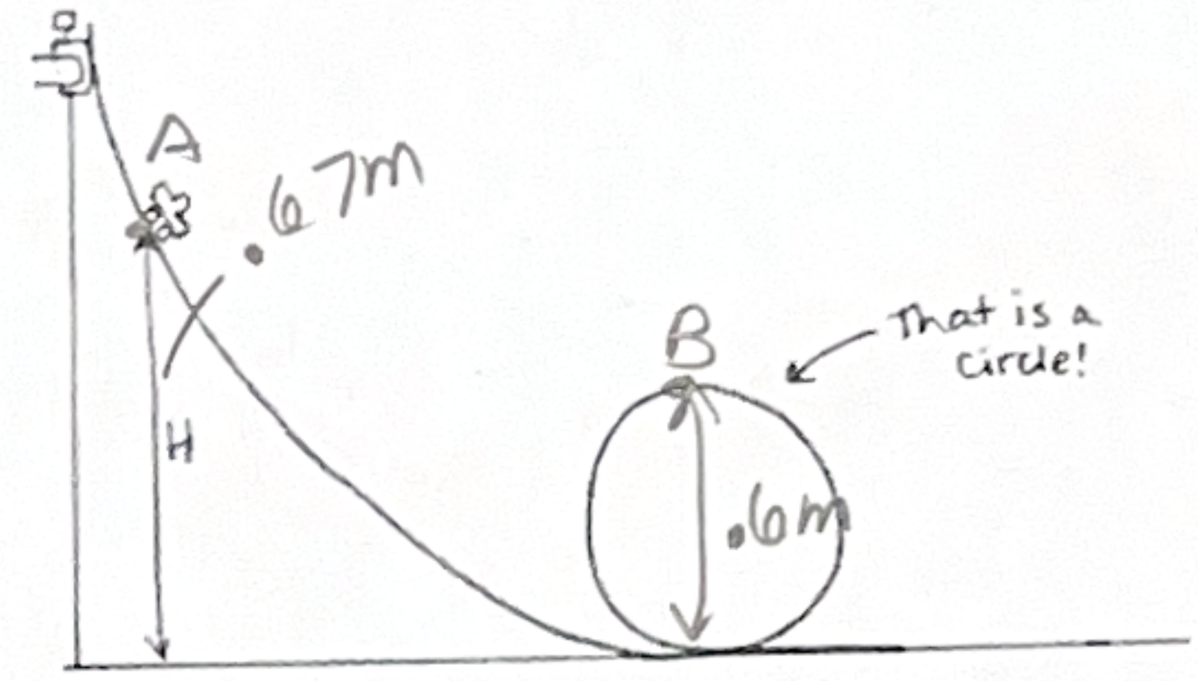
Name: ____________

Date: ____________

Work and Energy 7

1. A car of mass *m* is released from a height H of 67 cm above the floor on a track where friction is negligible. It rolls down the track and into a loop-the-loop which has a diameter of 0.6 m. Does the car make it around the loop-the-loop?

Your answer must be supported by calculations and any other sketches and diagrams that you find helpful. Also explain how you used the results of your calculations to answer the question.



To make it around the loop, the slowest it can go at B is when $\Sigma F_r = mg$, $\downarrow F_G$, because $\vec{n}=0$ at the slowest speed to make it. $a_r = \frac{\Sigma F}{m}$

$\frac{v^2}{r} = \frac{mg}{m}$

$\boxed{v=\sqrt{rg}}$

$v = \sqrt{(.3m)(10^N/_{kg})}$

$\boxed{v = 1.73\ m/s}$

Criteria statement: To make it around the loop, $v \geq 1.73$ m/s.

Now use energy to find the car's speed at B:

system: Earth, car, track

no ext forces do work

$E_i + \Sigma trans. = E_f$

$U_{GA} + 0 = U_{GB} + K_B$

$\cancel{m}gH = \cancel{m}gh_B + \frac{1}{2}\cancel{m}v_B^2$

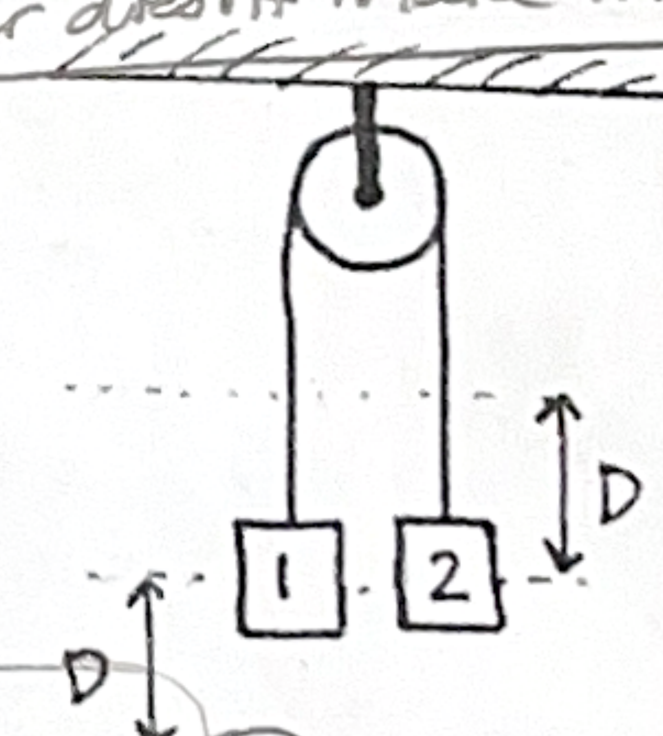
$\boxed{\sqrt{2(gH - gh_B)} = v_B}$

$v_B = \sqrt{2[(10N/kg)(.67m) - (10N/kg)(.6m)]}$

$\boxed{v_B = 1.18\ m/s}$

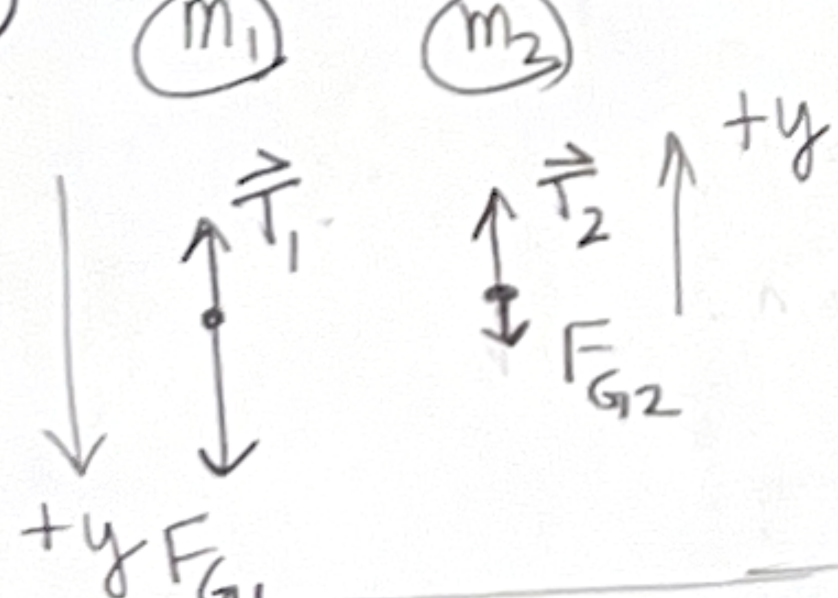
Evaluation: Because $v = 1.18$ m/s is less than $v = 1.73$ m/s the car doesn't make it.

2. An Atwood machine consists of block 1 (m_1) and block 2 (m_2) connected by a string that passes over an ideal pulley. $m_1 > m_2$. At the instant when m_1 has fallen a distance D, determine an expression for the common speed of the masses in terms of m_1, m_2, D, and fundamental constants, using each of the approaches below.



a. Newton's Laws: + kinematics *Include force diagrams and show as much work as needed for you to solve the problem and to clearly communicate your thought process.*

①



② For m_1: $a_1 = \frac{\Sigma F_{on 1}}{m_1}$

$a_1 = \frac{m_1 g - T_1}{m_1}$

$m_1 a_1 = m_1 g - T_1$

For m_2: $a_2 = \frac{\Sigma F_{on 2}}{m_2}$

$a_2 = \frac{T_2 - m_2 g}{m_2}$

$m_2 a_2 = T_2 - m_2 g$

Common variables: $a_1 = a_2 = a$, $T_1 = T_2 = T$

$\Rightarrow m_1 a = m_1 g - T$

$m_2 a = T - m_2 g$

add: $m_1 a = m_1 g - T$

$m_2 a = T - m_2 g$

$(m_1 + m_2)a = m_1 g - m_2 g$

$\boxed{a = \frac{g(m_1 - m_2)}{m_1 + m_2}}$

③ Now find Speed

$v_i = 0$

$v_f = ?$

$a =$ (that eqn I found)

$\Delta y = D$

$v_f^2 = v_i^2 + 2a\Delta y$

$v_f^2 = 0^2 + 2\left(\frac{g(m_1 - m_2)}{m_1 + m_2}\right) D$

$\boxed{v_f = \sqrt{\frac{2gD(m_1 - m_2)}{m_1 + m_2}}}$

positive because my +y direction is down: $\Delta y = +D$

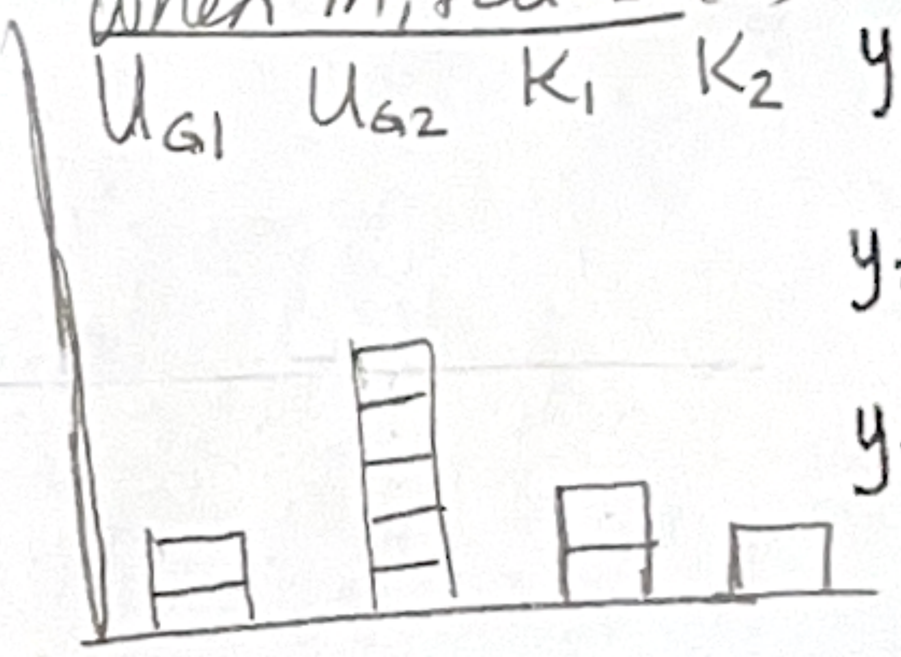
b. Energy *Include an energy bar chart in your analysis, using this system: m_1, m_2, string, pulley, Earth, and work that shows how you solved the problem.*

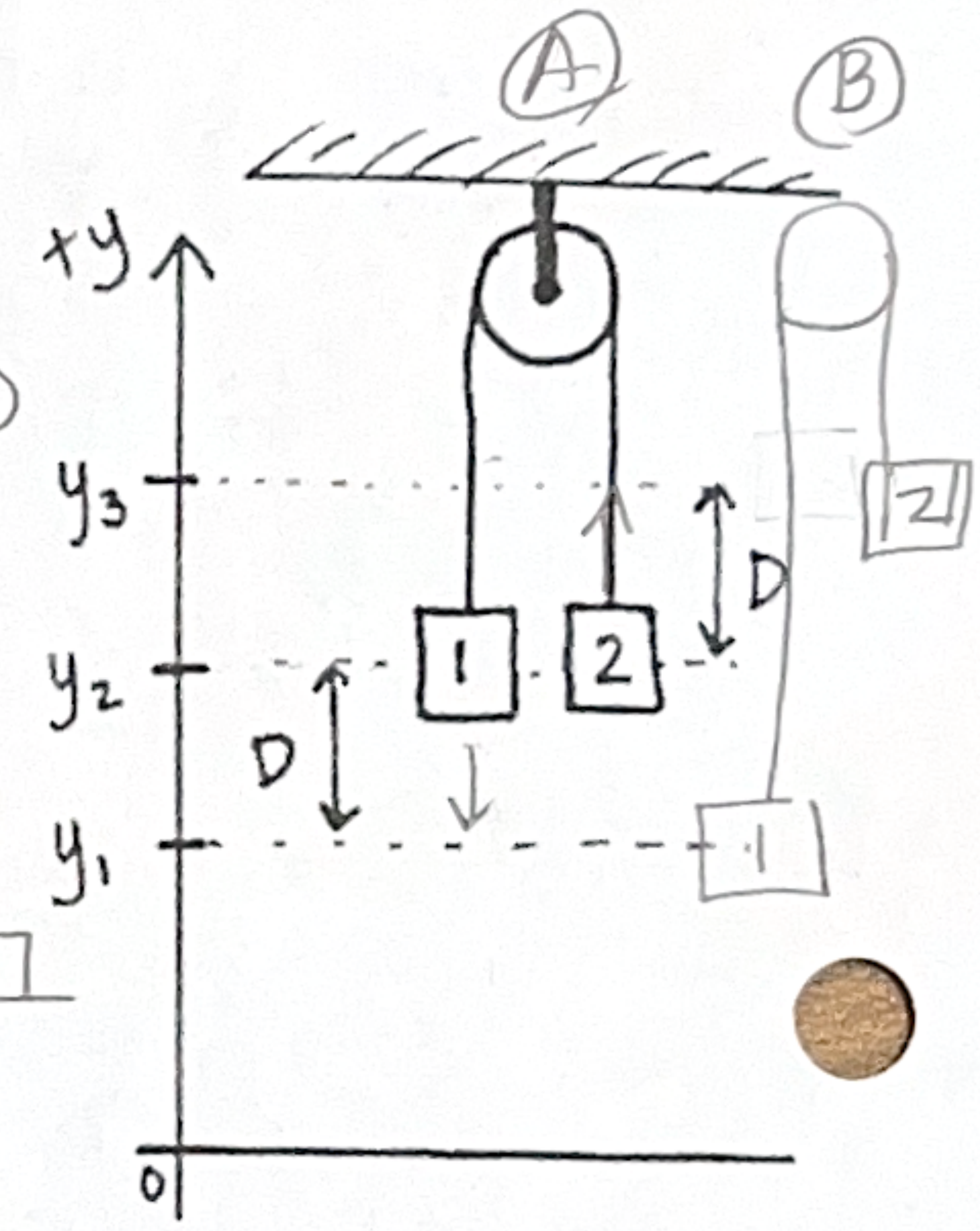
At rest (A)

m_1, m_2
string
pulley,
Earth

no transfers

When m_1 fell D (B)





m_1: U_G decreases, K increases

m_2: U_G increases, K increases

Since $m_1 > m_2$, $U_{G1} > U_{G2}$ at A ($U_G = mgy$)

At B, they have same speed, but m_1 has more K ($K = \frac{1}{2}mv^2$) $\quad V_1 = V_2 = V$

$E_i + \Sigma \text{transfers} = E_f$

$$U_{G1A} + U_{G2A} = U_{G1B} + U_{G2B} + K_{1B} + K_{2B}$$

$$m_1 g y_2 + m_2 g y_2 = m_1 g y_1 + m_2 g y_3 + \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2$$

$$\underbrace{m_1 g y_2 + m_2 g y_2 - m_1 g y_1 - m_2 g y_3}_{} = \tfrac{1}{2} v^2 (m_1 + m_2)$$

$$m_1 g (y_2 - y_1) + m_2 g (y_2 - y_3) = \tfrac{1}{2}(m_1 + m_2) v^2$$

$$m_1 g (+D) + m_2 g (-D) = \tfrac{1}{2}(m_1 + m_2) v^2$$

$$g D (m_1 - m_2) = \tfrac{1}{2}(m_1 + m_2) v^2$$

$$\frac{2 g D (m_1 - m_2)}{m_1 + m_2} = v^2$$

$$\boxed{\sqrt{\frac{2 g D (m_1 - m_2)}{m_1 + m_2}} = V}$$

Which matches the result I got using Newton's Laws + Kinematics!