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Name: ____________

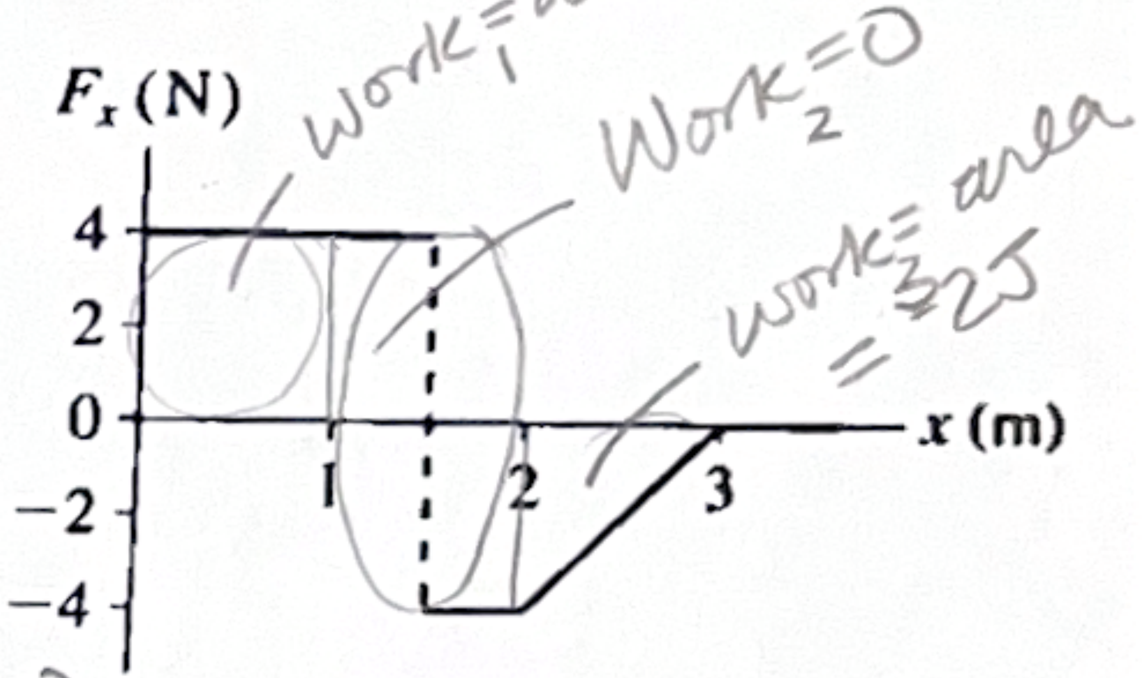
Date: ____________

Solutions posted

Work and Energy 6

Instructions: If using the conservation of energy, show the problem-solving steps, which include the external force and work analysis and the bar chart

1. A 5.0 kg particle moving along the x-axis experiences only the force shown in the graph. The particle's velocity at x = 0 m is +6.0 m/s. What is its velocity at x = 3 m? *(The force is not designated as a conservative force, so you cannot assume there is a potential energy associated with it. Source of graph: p. 228, Knight.)*



A (x = 0): $V_0 = +6$ m/s

B (x = 3 m)

At x = 0, v = 6 m/s (A)

At x = 3 m (B)

? K_f maybe

particle: W_1 = +4J, W_3 = −2J

From 0 → 1 m: $\vec{F}$ →, $\Delta\vec{r}$ → : W_1 is +

From 1 m → 2 m: F →, Δr → : +W; F ←, Δr ← : −W; $W_2 = 0$

From 2 m to 3 m: ← F, → Δr : W_3 is −

Ext Force + work Analysis

$$E_i + \Sigma \text{transfers} = E_f$$

$$K_A + W_1 + W_2 = K_B$$

$$\tfrac{1}{2}mV_A^2 + W_1 + W_2 = \tfrac{1}{2}mV_B^2$$

$$\sqrt{\tfrac{2}{m}\left(\tfrac{1}{2}mV_A^2 + W_1 + W_2\right)} = V_B$$

$$\sqrt{\tfrac{2}{5\text{kg}}\left[\tfrac{1}{2}(5\text{kg})(6\text{ m/s})^2 + 4\text{J} - 2\text{J}\right]} = V_B$$

$$\sqrt{\tfrac{2}{5\text{kg}}(90\text{J} + 2\text{J})} = V_B$$

$$\boxed{6.07\text{ m/s} = V_B}$$

$K_i = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(5)(6)^2 = 90$J

$W_{total} = +2$J

$K_f = 92$J

$v = 6.2$ m/s

Check: $E_i \Rightarrow K_i = \tfrac{1}{2}(5\text{kg})(6\text{ m/s})^2 = 90$J

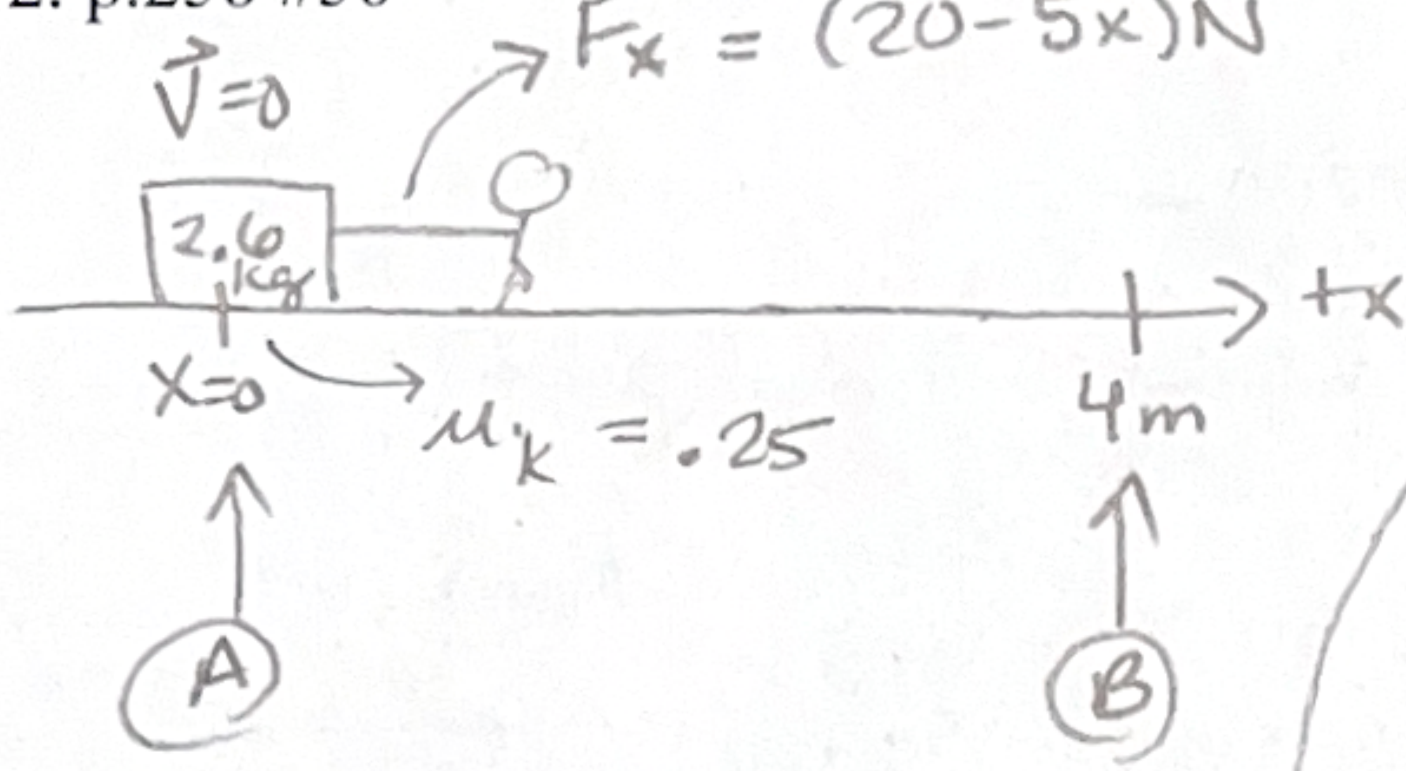
Σ transfers ⇒ +2J

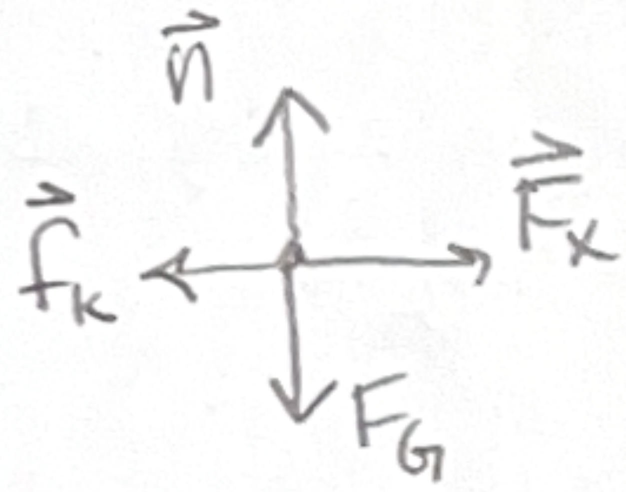
$E_f \Rightarrow K_f = \tfrac{1}{2}(5\text{kg})(6.07\text{ m/s})^2 = 92$J

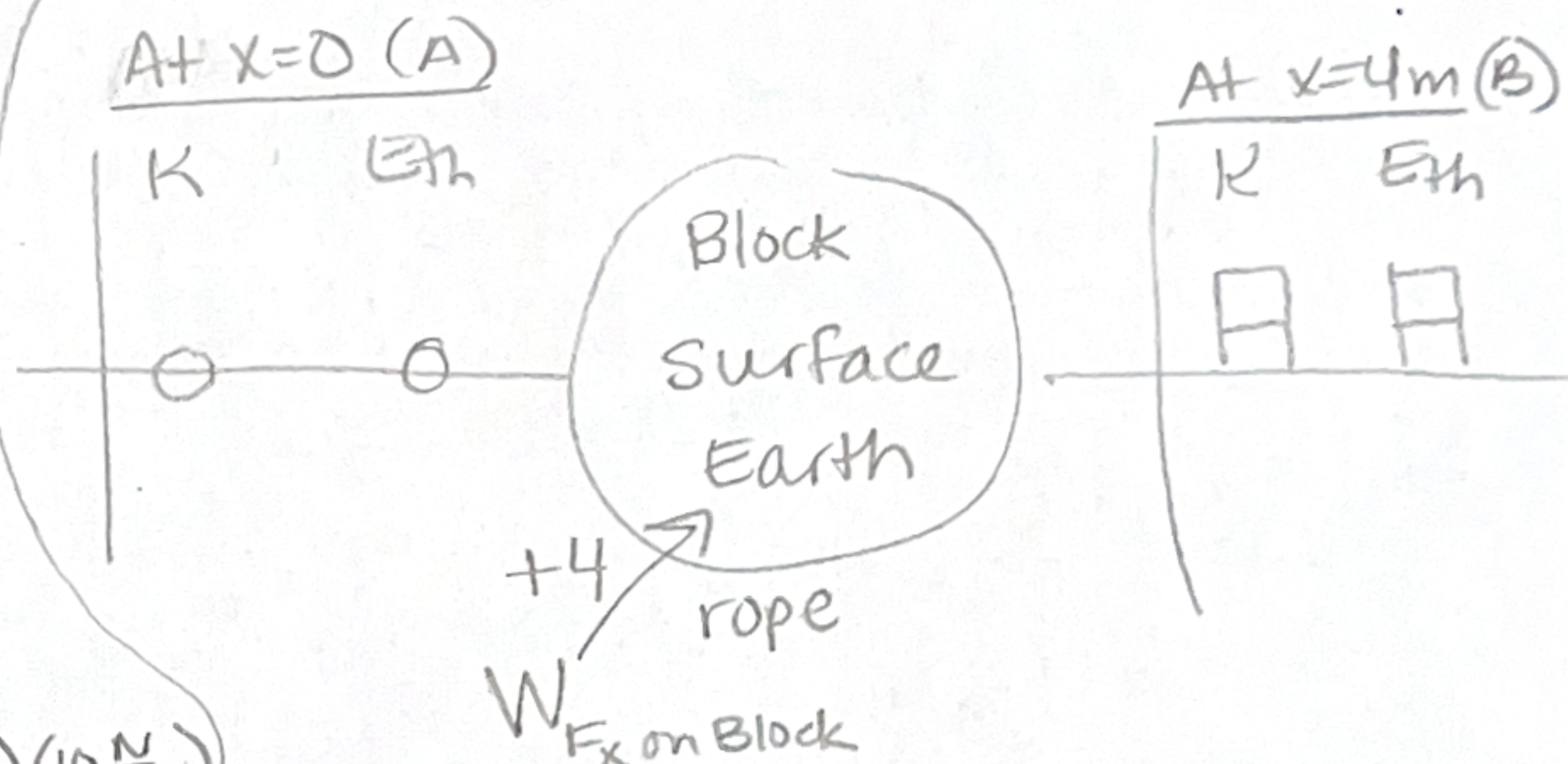
So yes, $E_i + \Sigma \text{trans} = E_f$

① Sketch + translate

2. p.258 #56







Kinetic friction: $f_k = \mu_k n$

$f_k = \mu_k mg$

$= (.25)(2.6kg)(10\frac{N}{kg})$

$= 6.5N$

② Simplify + Diagram

ext forces + work

$$\left.\begin{array}{l}\xrightarrow{\quad\vec{F}_x\quad} \\ \xrightarrow[\quad\Delta\vec{r}\quad]{}\end{array}\right\} W \text{ is } +$$

Calculate work

$$W = \int_a^b \vec{F} \cdot d\vec{r}$$

$$= \int_0^4 (20-5x)\hat{\imath} \cdot dx\,\hat{\imath}$$

$$= \int_0^4 (20-5x)\,dx$$

$$= \left[20x - \frac{5}{2}x^2\right]_0^4$$

$$= \left[20(4) - \frac{5}{2}(4)^2\right] - \left[20(0) - \frac{5}{2}(0)^2\right]$$

$$= 80 - 40 = 40J$$

③ Represent Mathematically + Solve

$E_i + \Sigma \text{ transfers} = E_f$

$$0 + W_{F_x \text{ on } B} = K_B + E_{th}$$

$$0 + W_{F_x \text{ on } B} = \frac{1}{2}mV_B^2 + f_k D$$

$$W_{F_x \text{ on } B} - f_k D = \frac{1}{2}mV_B^2$$

$$\sqrt{\frac{2}{m}\left[W_{F_x \text{ on } B} - f_k D\right]} = V_B$$

$$\sqrt{\frac{2}{2.6kg}\left[40J - (6.5N)(4m)\right]} = V_B$$

$$\sqrt{\frac{2}{2.6kg}(40J - 26J)} = V_B$$

$$\boxed{3.3\,m/s = V_B}$$