Solutions posted

Name: ____________

Date: ____________

Work and Energy 5

Instructions: Support your answers with mathematical or verbal explanations unless otherwise indicated.

1. New Facts: Write these into your booklet and memorize.

For a conservative force (like springs and F_G)
- The negative of the work done by a force from a to b (the negative of the area of the F vs. x graph from a to b) equals the change in potential energy: $-W = \Delta U$
- The negative derivative of the potential energy function (the negative of the slope of the U vs. x graph) is the force: $F = -\frac{dU}{dx}$

2. Finding change in potential energy and more from a F vs. x graph

Go to p.228 #20 and do these steps:

a. If the positive direction is to the right, what is the direction of the force on the particle at each of these positions? (No support needed)

x = 1 m right
x = 1.75 m left
x = 2.5 m left
x = 3.2 m (none)

b. Find the work during each interval as instructed in the problem.

Work done by F_x is the area of the graph

$0 \to 1\,m$: $W = (1m)(4N) = \boxed{4J}$

$1 \to 2\,m$: $W = (.5m)(4N) + (.5m)(-4N) = \boxed{0J}$

$2 \to 3\,m$: $W = \frac{1}{2}bh = \frac{1}{2}(1m)(-4m) = \boxed{-2J}$

c. What is the general relationship between the change in potential energy and the work done by a force? $-W = \Delta U$. State whether the potential energy is *increases, decreases* or *stays the same* during each interval given in the problem. Explain.

$0 \to 1\,m$: W is +, so ΔU is −, so U decreases

$1 \to 2\,m$: W is 0, so ΔU is 0, so U stays the same

$2 \to 3\,m$: W is −, so ΔU is +, so U increases

d. Find the change in potential energy during each interval given in the problem. Compare your answers here with your answers in **c**. → They are consistent.

$\Delta U = -W$, so

$0 \to 1\,m$: $\Delta U = \boxed{-4J}$

$1 \to 2\,m$: $\Delta U = \boxed{0}$

$2 \to 3\,m$: $\Delta U = \boxed{+2J}$

3. Finding change in potential energy and a potential energy function from a force function

A nonlinear spring exerts a force $F = -kx^2$ when displaced to position x from equilibrium.

$\vec{F} = -kx^2\hat{\imath}$

a. Determine the change in potential energy when this spring is displaced from x_i to x_f.

$$-W = \Delta U$$

$$-\int_a^b \vec{F}\cdot d\vec{r} = \Delta U$$

$$-\int_{x_i}^{x_f} (-kx^2\hat{\imath})\cdot dx\hat{\imath} = \Delta U$$

$$\int_{x_i}^{x_f} kx^2 dx = \Delta U \rightarrow k\left(\tfrac{1}{3}x^3\right)\Big]_{x_i}^{x_f} = \Delta U$$

$$\boxed{\tfrac{1}{3}kx_f^3 - \tfrac{1}{3}kx_i^3 = \Delta U}$$

b. Determine the potential energy function associated with this spring.

$$\tfrac{1}{3}kx_f^3 - \tfrac{1}{3}kx_i^3 = U_f - U_i$$

$$\boxed{U = \tfrac{1}{3}kx^3}$$

4. Finding the force and more from a U vs. x graph

Use just the graph from #28 on p.257 to answer these questions. Assume the positive direction is to the right.

a. Calculate the force on the particle at x = 2m and x = 5 m. State the direction of the force at each position.

This is a U vs. x graph, and $F = -\frac{dU}{dx}$, so force is the negative of the slope of the U graph.

At x=2m, slope = $4J/2m = 2\,J/m = 2\,Nm/m = 2N$. The negative of the slope is $\boxed{-2N}$ $\boxed{\text{Left}}$

At x=5m, Force = −Slope = $-\left(\frac{-8J}{2m}\right) = \boxed{+4N}$ – $\boxed{\text{Right}}$

b. What direction would the particle would move if it was released from rest at x = 2 m? x = 5 m? (No support needed).

at x=2m, $\vec{F}$ is to the left, so if released from rest, $\boxed{\text{it would speed up to the left}}$

at x=5m, $\vec{F}$ is to the right, so if released from rest there, $\boxed{\text{it would speed up to the right}}$

c. What is the sign of the work done by the force on the particle from x = 1m to x = 4 m? Force is −, Δr is +, so $\boxed{W \text{ is } -}$

Reading the graph, does the potential energy increase or decrease during this displacement? increase. Is this consistent with the idea that $\Delta U = -W$? yes, W is − and ΔU is + (No support needed).

d. Calculate the change in potential energy if the particle moves from x = 2 m to x = 8 m.

$$\Delta U = U_f - U_i$$

$$= 12J - 4J$$

$$\boxed{+8J}$$

5. Finding the force from a potential energy function and more

A system's potential energy is given by $U = (2x^3 - 3x^2)$ J, where x is a particle's position in m.

a. What is the potential energy of the system when the particle is at x = 1 m?

U at x=1m:

$$U = (2x^3 - 3x^2)\,J$$
$$= (2(1)^3 - 3(1)^2)\,J$$
$$= (2-3)\,J = \boxed{-1J}$$

b. Calculate the change in potential energy as the particle moves from x = 1 m to x = 2 m.

Find U at x=2m first: $U = (2(2)^3 - 3(2)^2)\,J$
$$= (16 - 12)\,J$$
$$= 4J$$

Find ΔU: $\Delta U = U_f - U_i$
$$= 4J - (-1J)$$
$$= \boxed{5J}$$

c. Find an expression for the force associated with this potential energy function.

$$F = -\frac{dU}{dx}$$
$$F = -\frac{d}{dx}(2x^3 - 3x^2)$$
$$F = -(6x^2 - 6x)$$
$$\boxed{F = -6x^2 + 6x}$$

d. Calculate the force on the particle at x = 2 m. If the positive direction is to the right, what is the direction of this force?

$$F(2m) = -6(2)^2 + 6(2)$$
$$= -24 + 12$$
$$= \boxed{12N} \rightarrow \boxed{Right}$$