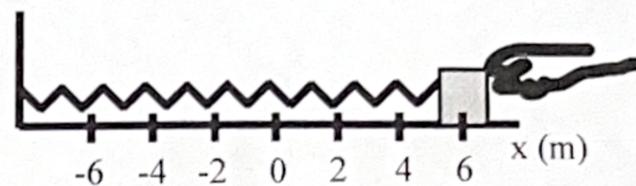


Work and Energy 4

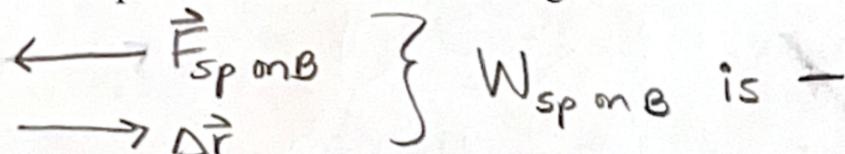
1. Physics facts: Copy the new facts into your booklet. Have all the mathematical relationships memorized by tomorrow, including any you missed on the recent quiz.

2. Work done by the spring force

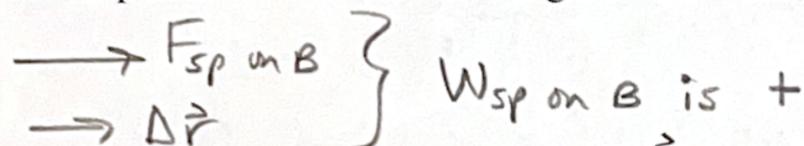
A block is attached to a horizontal spring with spring constant $k = 10 \text{ N/m}$. The equilibrium position for the spring is at $x = 0$.



a. The block has a displacement is from $x_i = 2 \text{ m}$ to $x_f = 6 \text{ m}$. Sketch vectors representing the spring force and the displacement. What is the sign of the work done by the spring on the block, $W_{sp \text{ on } B}$?



b. The block has a displacement is from $x_i = -6 \text{ m}$ to $x_f = 0 \text{ m}$. Sketch vectors representing the spring force and the displacement. What is the sign of the work done by the spring on the block, $W_{sp \text{ on } B}$?



c. What is the spring force written as a vector? $\vec{F}_{sp} = (-kx)\hat{i}$. As the block attached to the spring undergoes a displacement, is the force exerted by the spring on the block constant or varying? varying. What equation is used to find the work done by a varying force? $W = \int_a^b \vec{F} \cdot d\vec{r}$

d. A person pulls the block to cause a displacement from x_i to x_f . Derive an expression for the work done by the spring on the block as the block is displaced from x_i to x_f . Your answer will be in terms of k, x_i , and x_f .
 $d\vec{r}$ is a bit of displacement along the x-axis, so $d\vec{r} = (dx)\hat{i}$

$$W_{sp} = \int_a^b \vec{F} \cdot d\vec{r}$$

$$W_{sp} = \int_{x_i}^{x_f} (-kx)\hat{i} \cdot dx\hat{i}$$

$$W_{sp} = \int_{x_i}^{x_f} -kx dx$$

$$W_{sp} = \left[-\frac{kx^2}{2} \right]_{x_i}^{x_f}$$

$$W_{sp} = \left(-\frac{kx_f^2}{2} - -\frac{kx_i^2}{2} \right)$$

$$W_{sp} = -\frac{kx_f^2}{2} + \frac{kx_i^2}{2} = \boxed{\frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2}$$

e. Use the equation you derived in d. to calculate the work done by the spring during the displacement described in a. Does the sign for the work done by the spring match your prediction? yes!

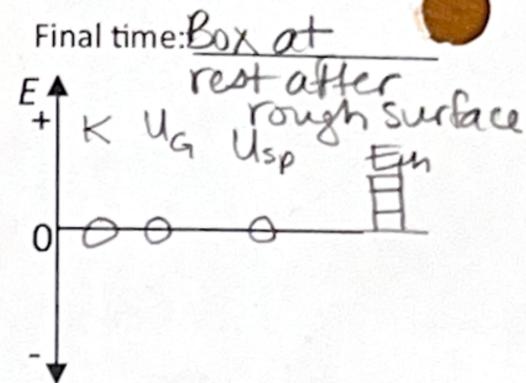
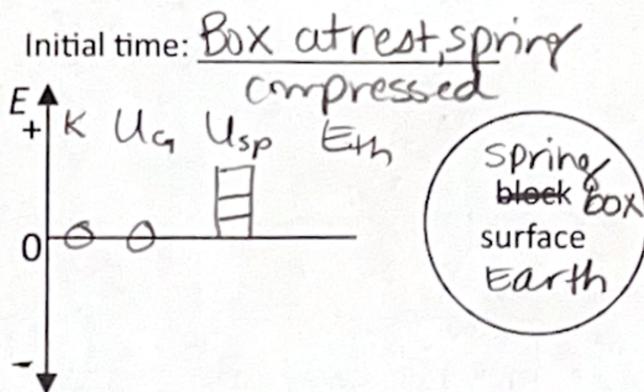
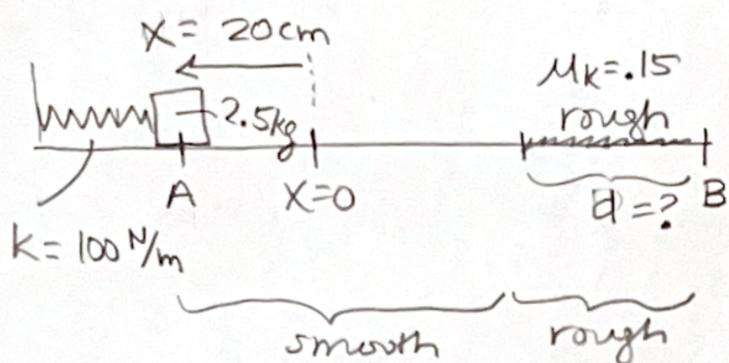
$$W_{sp \text{ on } B} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(10 \frac{\text{N}}{\text{m}})(2\text{m})^2 - \frac{1}{2}(10 \frac{\text{N}}{\text{m}})(6\text{m})^2 = 20 \text{ Nm} - 180 \text{ Nm}$$

$$= 20 \qquad \qquad \qquad = \boxed{-160 \text{ J}}$$

f. Use the equation you derived in d. to calculate the work done by the spring during the displacement described in b. Does the sign for the work done by the spring match your prediction? yes!

$$W_{sp \text{ on } B} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(10 \frac{\text{N}}{\text{m}})(-6\text{m})^2 - \frac{1}{2}(10 \frac{\text{N}}{\text{m}})(0)^2 = \boxed{+180 \text{ J}}$$

3. p.258 #49 - Horizontal spring and rough surface. Show the problem-solving steps.



- no ext forces
- no work transfers

$$E_i + \Sigma \text{transf.} = E_f$$

$$U_{sp} = E_{th}$$

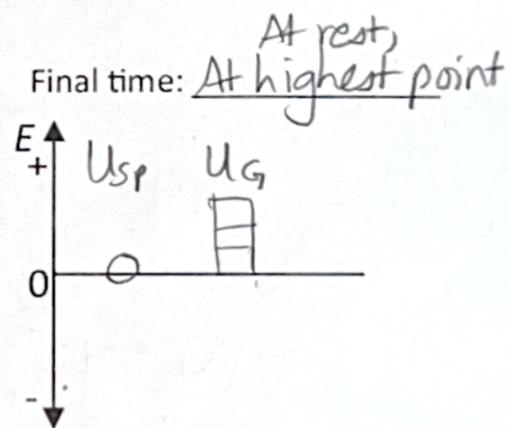
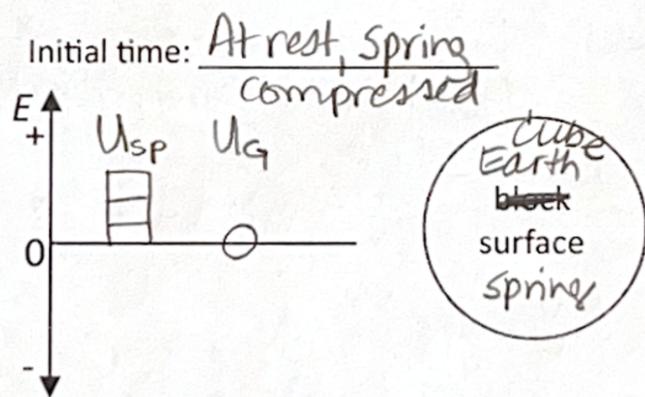
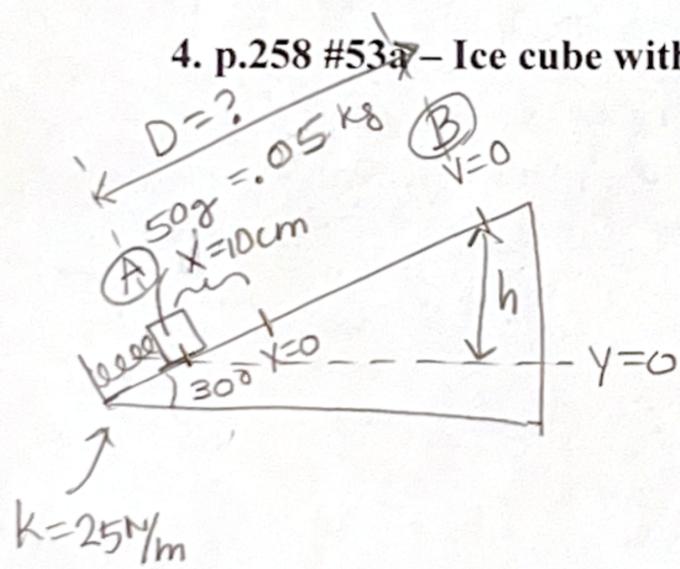
$$\frac{1}{2}kx^2 = f_k d$$

$$\frac{1}{2}kx^2 = \mu_k mgd$$

$$d = \frac{kx^2}{2\mu_k mg}$$

$$= \frac{(100 \text{ N/m})(.2 \text{ m})^2}{2(.15)(2.5 \text{ kg})(10 \text{ N/kg})} = \boxed{.53 \text{ m}}$$

4. p.258 #53a - Ice cube with spring on incline. Show the problem-solving steps.



- no ext. forces
- no work transfers

$$E_i + \Sigma \text{transfers} = E_f$$

$$U_{spA} = U_{gB}$$

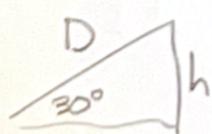
$$\frac{1}{2}kx^2 = mgh$$

$$h = \frac{kx^2}{2mg}$$

$$h = \frac{(25 \text{ N/m})(.10 \text{ m})^2}{2(.05 \text{ kg})(10 \text{ N/kg})}$$

$$h = .25 \text{ m} \text{ This is just the vertical height at B.}$$

But I need D:



$$\sin 30^\circ = \frac{h}{D}$$

$$D = \frac{h}{\sin 30^\circ}$$

$$D = \frac{.25 \text{ m}}{\sin 30^\circ} = \boxed{.5 \text{ m}}$$