Solutions posted

Name: ____________

Date: ____________

Work and Energy 1

1. Memory check: Write these relationships from memory. Then use your physics facts to check your work and fill in the ones you didn't know. Keep memorizing!

$E_i + \Sigma trans = E_f$	The law of conservation of energy (either form)
$U_G = mgy$	Gravitational potential energy, near a planet
$K = \frac{1}{2}mv^2$	Translational kinetic energy
$\vec{A}\cdot\vec{B} = AB\cos\alpha$	The dot product $\vec{A}\cdot\vec{B}$
$W = \vec{F}\cdot\Delta\vec{r}$	Work done by a constant force
$W = \int_a^b \vec{F}\cdot d\vec{r}$	Work done by a constant or variable force

2. p.227 CONCEPTUAL QUESTION #3

$(F_G > T)$

Tension: $\uparrow\vec{T}$ $\uparrow\Delta\vec{r}$ The $\vec{F}$ and $\Delta\vec{r}$ are in the same direction, so W is +

Gravity: $\downarrow\vec{F}_G$ $\uparrow\Delta\vec{r}$ The $\vec{F}$ and $\Delta\vec{r}$ are in the opposite direction, so W is −

3. p.228 #20

a. Do the problem as posed in the text:

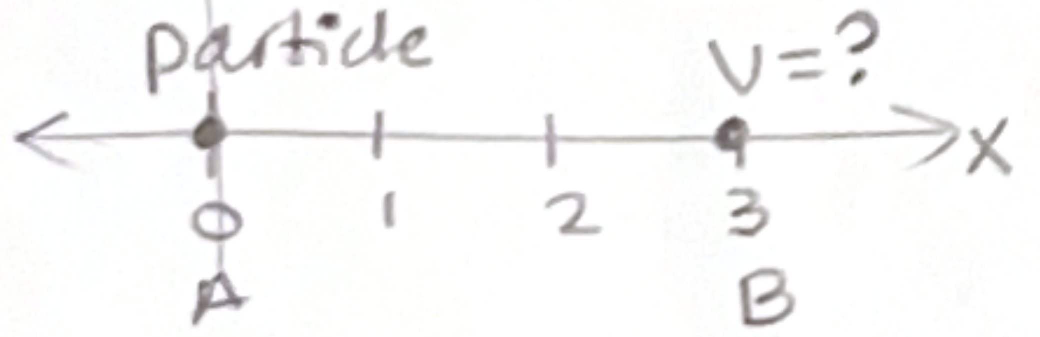
Area is Work: $0 \to 1m: W_1 = area = (4N)(1m) = 4J$

$1m \to 2m: W_2 = area = 0J$

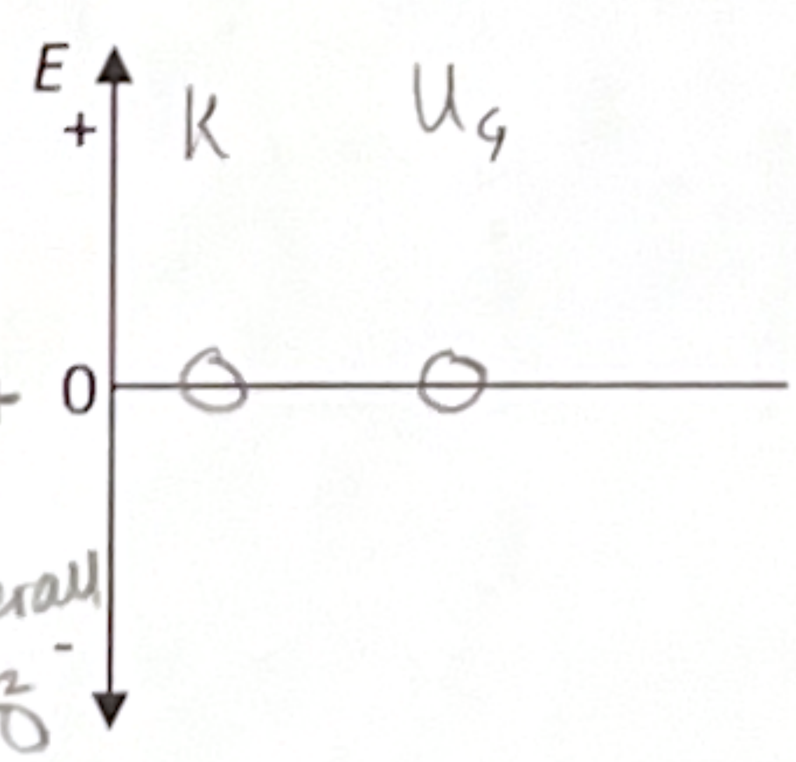
$2m \to 3m: W_3 = area = (\frac{1}{2})(1.0m)(-4N) = -2J$

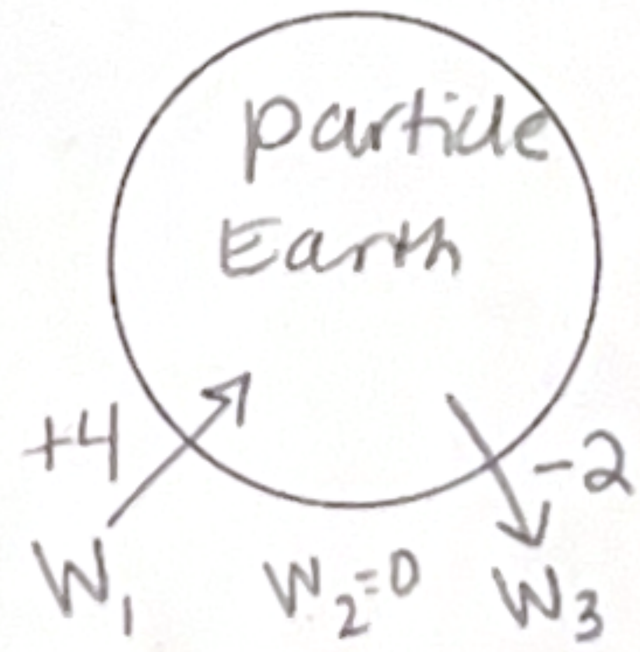
b. If the particle has a mass of 3 kg, and it is at rest at x = 0 m, use energy conservation to find its speed when it is at x = 3 m. Show the steps of the problem-solving strategy. (For the external force and work analysis, do each ~~segment~~ displacement of the graph separately, and show the work done on the system during each ~~segment~~ as a separate qualitatively-correct transfer on the bar chart.)

$m = 3kg$ particle

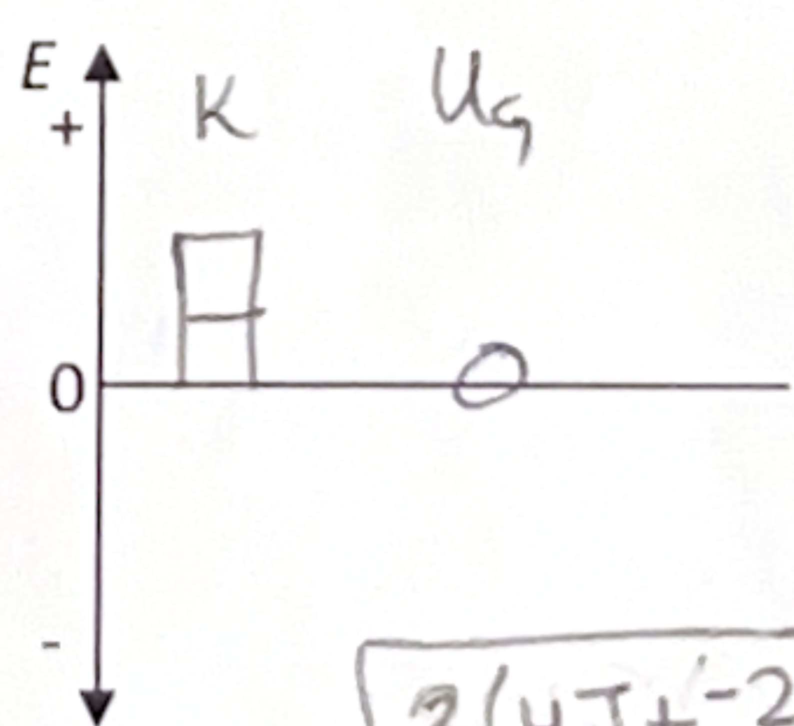


Initial time: Particle at x=0m (A)





Final time: particle at x=3m (B)



From 0m→1m: $\vec{F}$ → $\Delta\vec{r}$ → W_1 is +

From 1m→2m: First: $\vec{F}$ → $\Delta\vec{r}$ → +W; Then: $\vec{F}$ ← $\Delta\vec{r}$ → −W; overall W_2 is 0

From 2m→3m: $\vec{F}$ ← $\Delta\vec{r}$ → W_3 is −

$E_i + \Sigma trans. = E_f$

$0 + W_1 + W_3 = K_B$

$W_1 + W_3 = \frac{1}{2}mv_B^2$

$\sqrt{\frac{2(W_1+W_3)}{m}} = v_B$

$v_B = \sqrt{\frac{2(4J + -2J)}{3kg}}$

$= \boxed{1.2 m/s}$

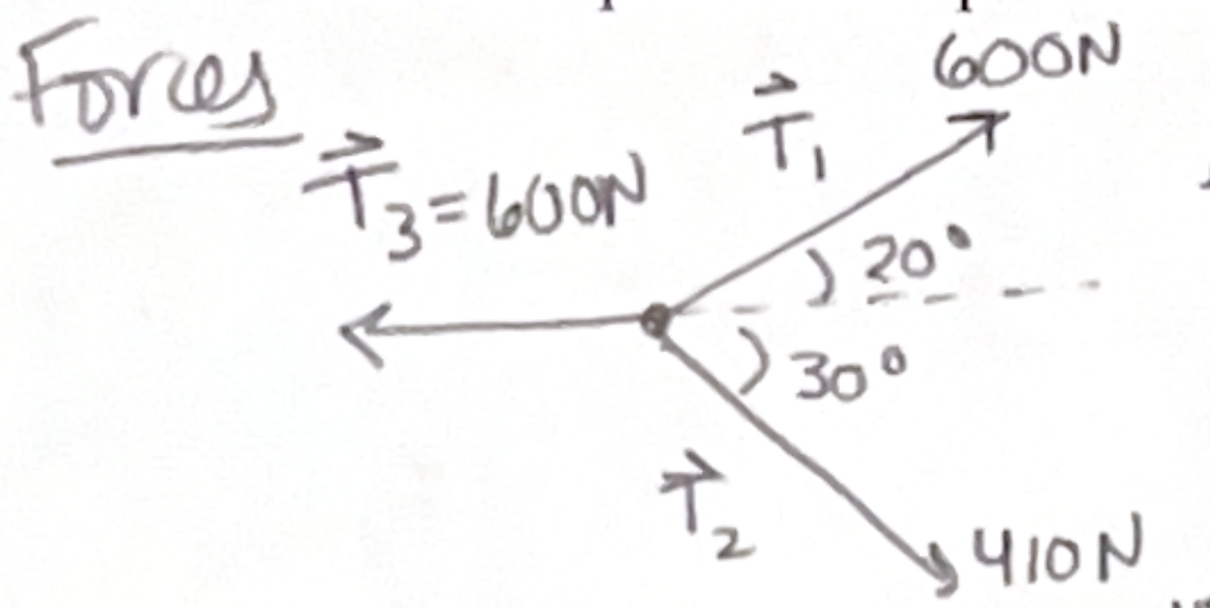
$W_1 = \vec{T}_1 \cdot \Delta\vec{r} = (T_1)(\Delta r)\cos\alpha$

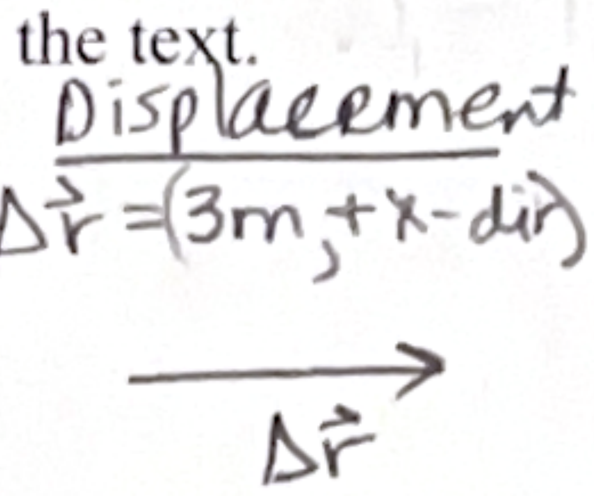
$W = (600N)(3m)\cos 20°$

$W = \boxed{1691 J}$

4. p.228 #19

a. Do the problem as posed in the text.





$W_2 = \vec{T}_2 \cdot \Delta\vec{r} = (T_2)(\Delta r)\cos\alpha$

$= (410N)(3m)\cos 30°$

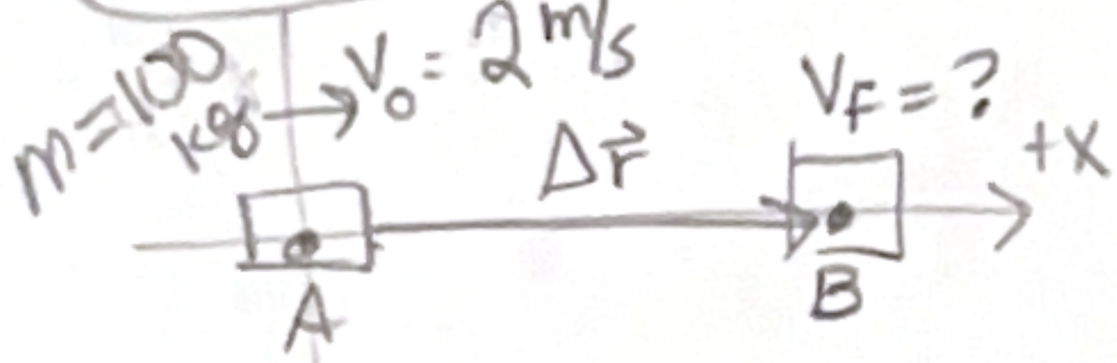
$= \boxed{1065 J}$

$W_3 = \vec{T}_3 \cdot \Delta\vec{r} = (T_3)(\Delta r)\cos\alpha$

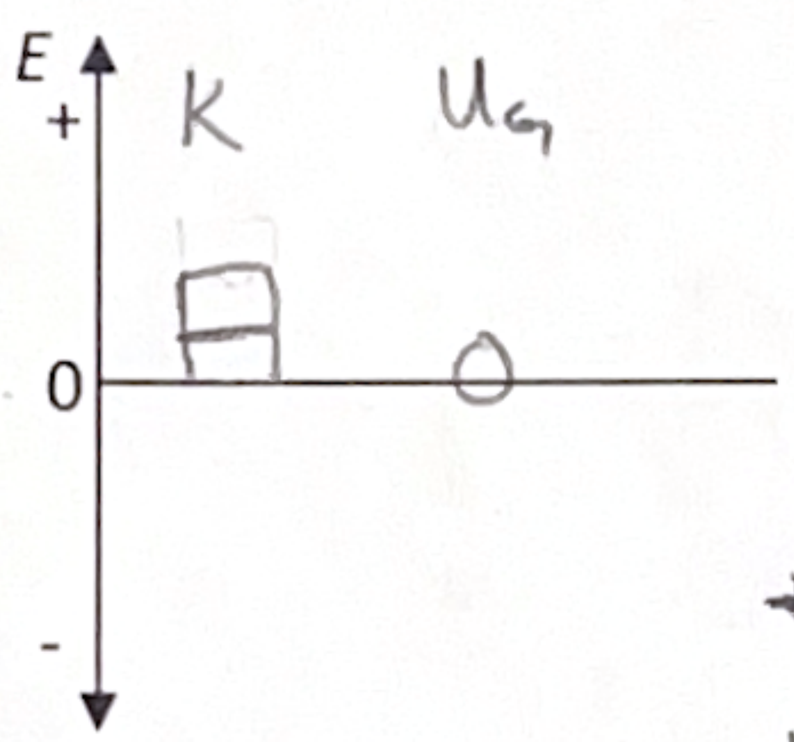
$= (600 N)(3m)\cos 180° = \boxed{-1800 J}$

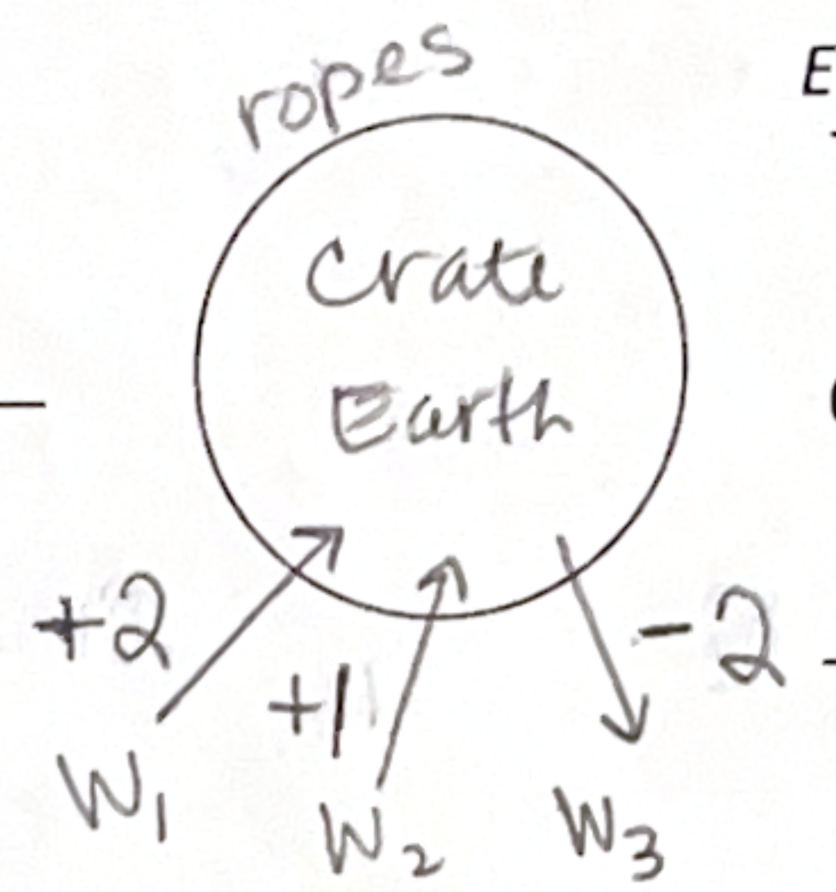
b. If the crate has a mass of ~~10~~ 100 kg, and it was moving at a speed of ~~1~~ 2 m/s when the ropes began to pull on it, find the speed of the crate after it had been pulled the displacement of 3 m. Use energy conservation and show the problem-solving steps. (Do the external force and work analysis for each rope separately, and show the work done on the system during each segment as a separate qualitatively-correct transfer on the bar chart.)

aerial view

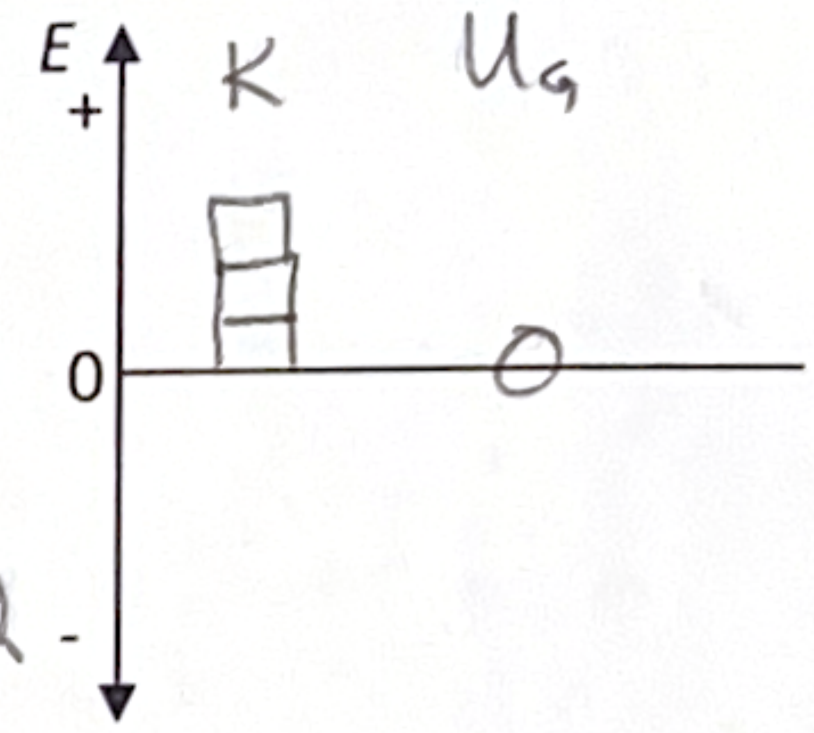


Initial time: moving at 1 m/s





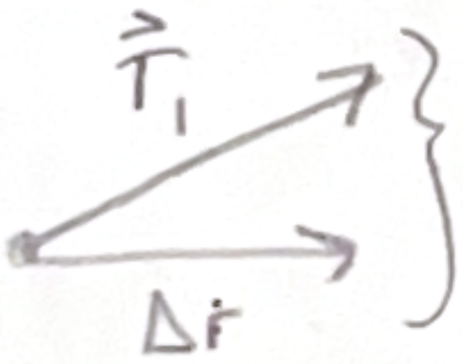
Final time: after 3m displacement



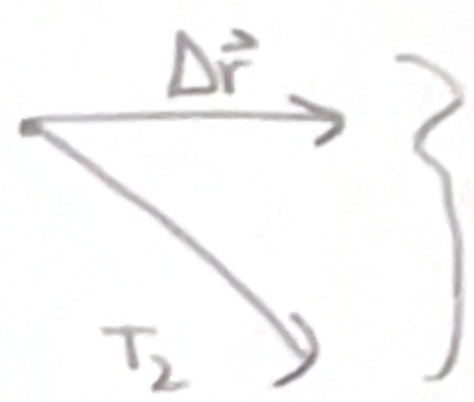
Crate is at y=0 the whole time, so $U_G = 0$.

I know from (a) that more energy goes in than out, so I chose numbers here that could represent that.

External forces: $\vec{T}_1, \vec{T}_2, \vec{T}_3$



W_1 is +



W_2 is +

$E_i + \Sigma\,\text{transfers} = E_f$

$K_A + W_1 + W_2 + W_3 = K_B$

$\frac{1}{2}mv_A^2 + W_1 + W_2 + W_3 = \frac{1}{2}mv_B^2$

W_3 is −

$\sqrt{\frac{2}{m}\left(\frac{1}{2}mv_A^2 + W_1 + W_2 + W_3\right)} = V_B$

$\sqrt{\frac{2}{100kg}\left(\frac{1}{2}(100kg)(2\frac{m}{s})^2 + 1691J + 1065J - 1800J\right)} = V_B$

$\sqrt{\frac{1}{50}(200 + 956)} = V_B$

$\boxed{4.8\,m/s = V_B}$