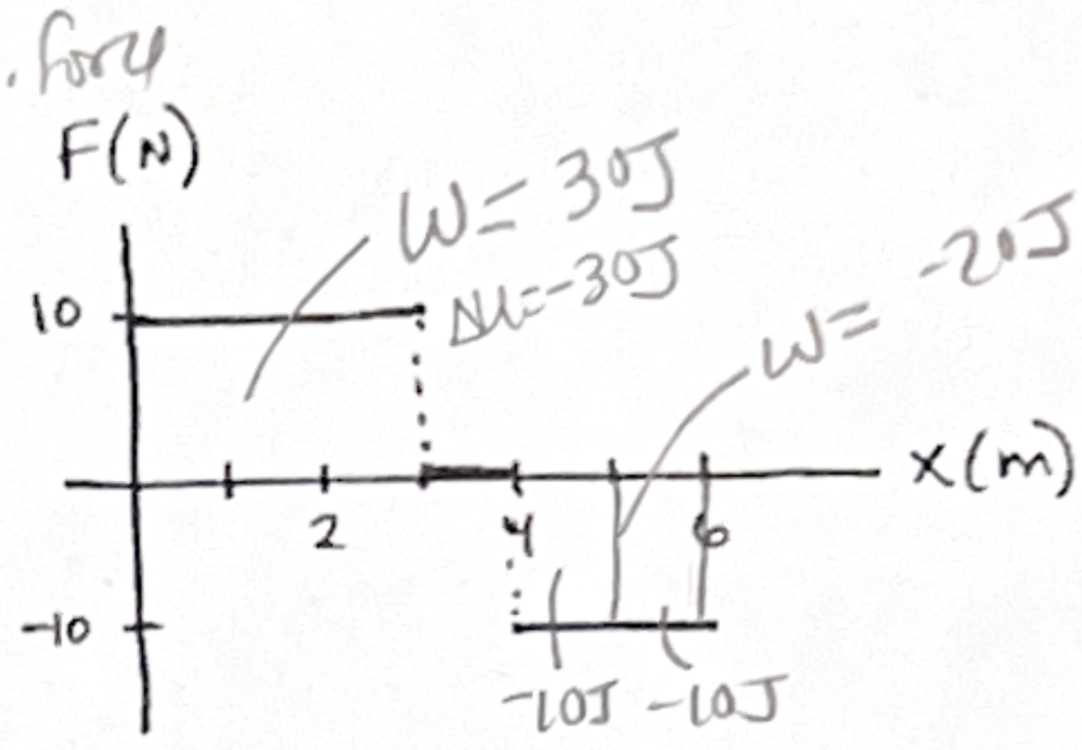
Name: ____________

Unit 5 Last Class

1. A 2 kg particle is in a system with ~~other~~ another objects which exerts a cons. force on it. This graph shows the ~~conservative~~ force which is conservative on the particle. The positive direction is to the right.

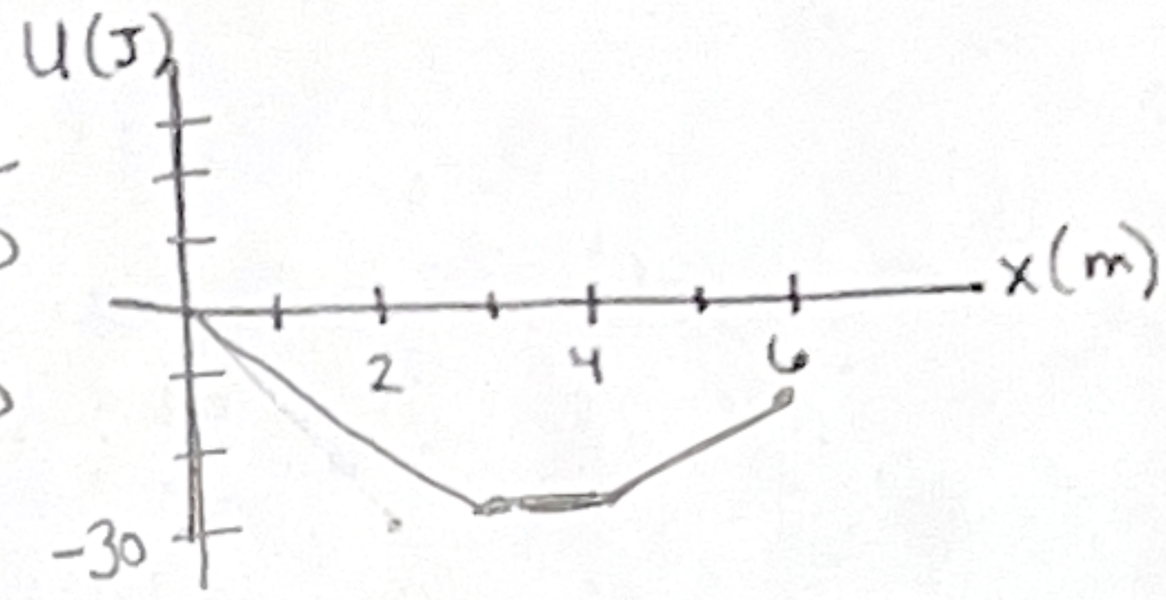


a. Find the change in potential energy as the particle moves from x = 0 to x = 5 m. $\Delta U = -W$. W = area

$\Delta U = -(30J - 10J) = -20J$

b. Find the work done on the particle from x = 0 m to x = 5 m.

$W = +20J$



Let U=0 at x=0

c. What is the direction of the force on the particle at x = 5 m? Determine a value for this force. Left

F = 10N, left

d. Calculate the speed of the particle at x = 5 m, if it had a velocity of + 3 m/s when it was at x = 0 m. $E_i + \text{transfers} = E_f$

$K_i + 20J = K_f$

$\frac{1}{2}m(3)^2 + 20J = \frac{1}{2}(2)v_f^2$

$v = 5.4 \text{ m/s}$

$U_i + K_i = U_f + K_f$

$0 = \Delta U + \Delta K$

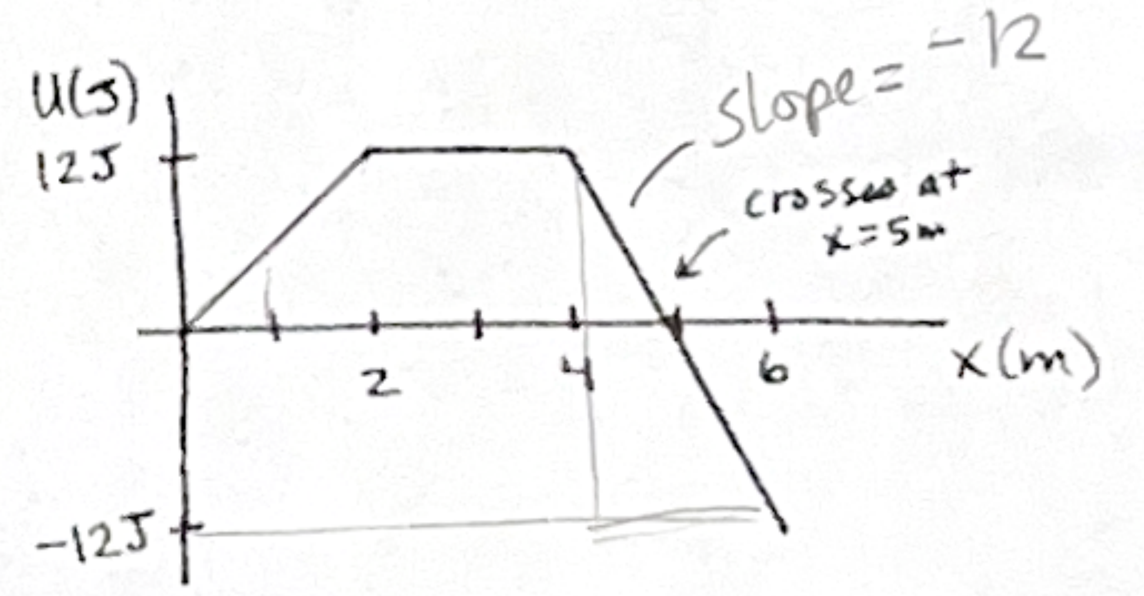
$0 = -20J + \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$20J = \frac{1}{2}(2kg)(v_f^2) - \frac{1}{2}(2kg)(3)^2$

$v = 5.4 \text{ m/s}$

e. Sketch the graph of the potential energy as a function of position on the axes above. Include values on the y-axis.

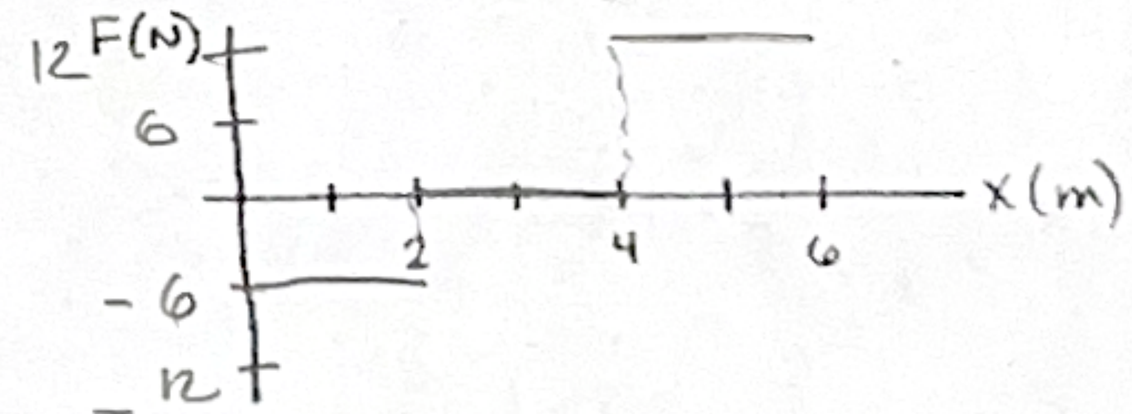
2. This graph represents the potential energy of a system as function of the position of a 3 kg particle in the system. The positive direction is to the right.



a. Find the change in potential energy as the particle moves from x = 0 to x = 6 m. $\Delta U = U_f - U_i = -12J - 0 = -12J$

b. Find the work done on the particle from x = 0 to x = 6 m.

$-W = \Delta U$, so $W = +12J$



c. What is the direction of the force on the particle at x = 1 m? Determine a value for this force. F = − slope of U graph

$F = -(6 \text{ J/m}) = -6N$ left, 6N

d. Sketch a graph of the force as a function of position on the axes above.

e. Calculate the speed of the particle at x = 6 m, if it had a velocity of + 4 m/s at x = 0.

X=0 to x=6: $E_i + E_{trans} = E_f$

$U_i + K_i = U_f + K_f$

$0J + \frac{1}{2}mv^2 = -12J + \frac{1}{2}mv^2$

$\frac{1}{2}(3kg)(4 \text{ m/s})^2 = -12J + \frac{1}{2}(3kg)v^2$

$24 + 12 = \frac{3}{2}v^2$

$v = 4.9 \text{ m/s}$

makes sense b/c U decreased, so K increased

f. What is the acceleration of the particle at x = 1m?

$a = \frac{F}{m} = \frac{-6N}{3kg} = -2 \text{ m/s}^2$

g. How much time did it take the particle to move from 0 m to 2 m? Need v at x=2m:

$E_i + \Sigma\text{transfers} = E_f$

$K_i = U_f + K_f$

$\frac{1}{2}(3kg)(4 \text{ m/s})^2 = 12J + \frac{1}{2}(3kg)(v_f)^2$

$24J = 12J + 1.5 v_f^2$

$2.8 \text{ m/s} = v_f$

$a = -2 \text{ m/s}^2$

$v_i = 4 \text{ m/s}$

$v_f = 2.8 \text{ m/s}$

$\Delta t = ?$

$v_f = v_i + a\Delta t$

$2.8 \text{ m/s} = 4 \text{ m/s} + (-2 \text{ m/s}^2)\Delta t$

$\boxed{.6s} = \Delta t$

3.

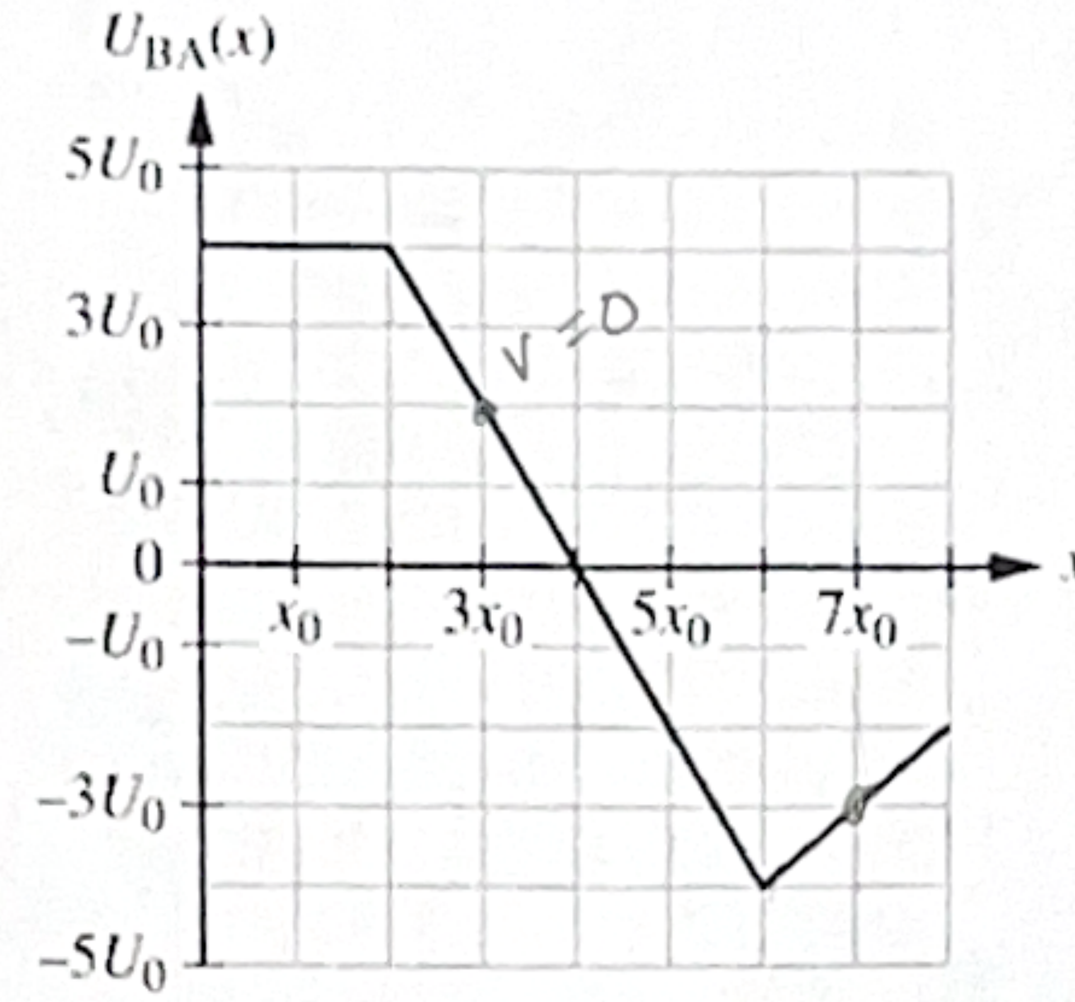


Figure 1

A system is comprised of two objects, A and B, which interact with each other through a conservative force $F_{BA}(x)$, where x is the position of Object A with respect to Object B. The potential energy $U_{BA}(x)$ of the two-object system as a function of x is shown in Figure 1. No external forces are exerted on the two-object system.

initial: At $3x_0$
final: At $7x_0$

$E_i + \Sigma trans = E_f$

$U_i + 0 + 0 = U_f + K_f$

$2U_0 = -3U_0 + \frac{1}{2} m_A v^2$

$\frac{5U_0(2)}{m_A} = v^2$

$V = \sqrt{\frac{10U_0}{m_A}}$

Object A has mass m_A and is released from rest at position $x = 3x_0$. Object A starts moving, and later passes through position $x = 7x_0$.

(a)

i. **Derive** an expression for the speed of Object A at the instant Object A is passing through position $x = 7x_0$. Express your answer in terms of m_A, U_0, x_0, and physical constants, as appropriate.

ii. On the grid shown in Figure 2, **draw** a graph of the force exerted on Object A by Object B.

F = -slope of U graph

$0 \to 2x_0$, slope = 0, F = 0

$2x_0 \to 6x_0$, slope $= -\frac{8U_0}{4x_0}$; $F = +2\frac{U_0}{x_0}$

$6x_0 \to 8x_0$, slope $= +\frac{2U_0}{2x_0}$, $F = -\frac{U_0}{x_0}$

Let $F_0 = +\frac{U_0}{x_0}$

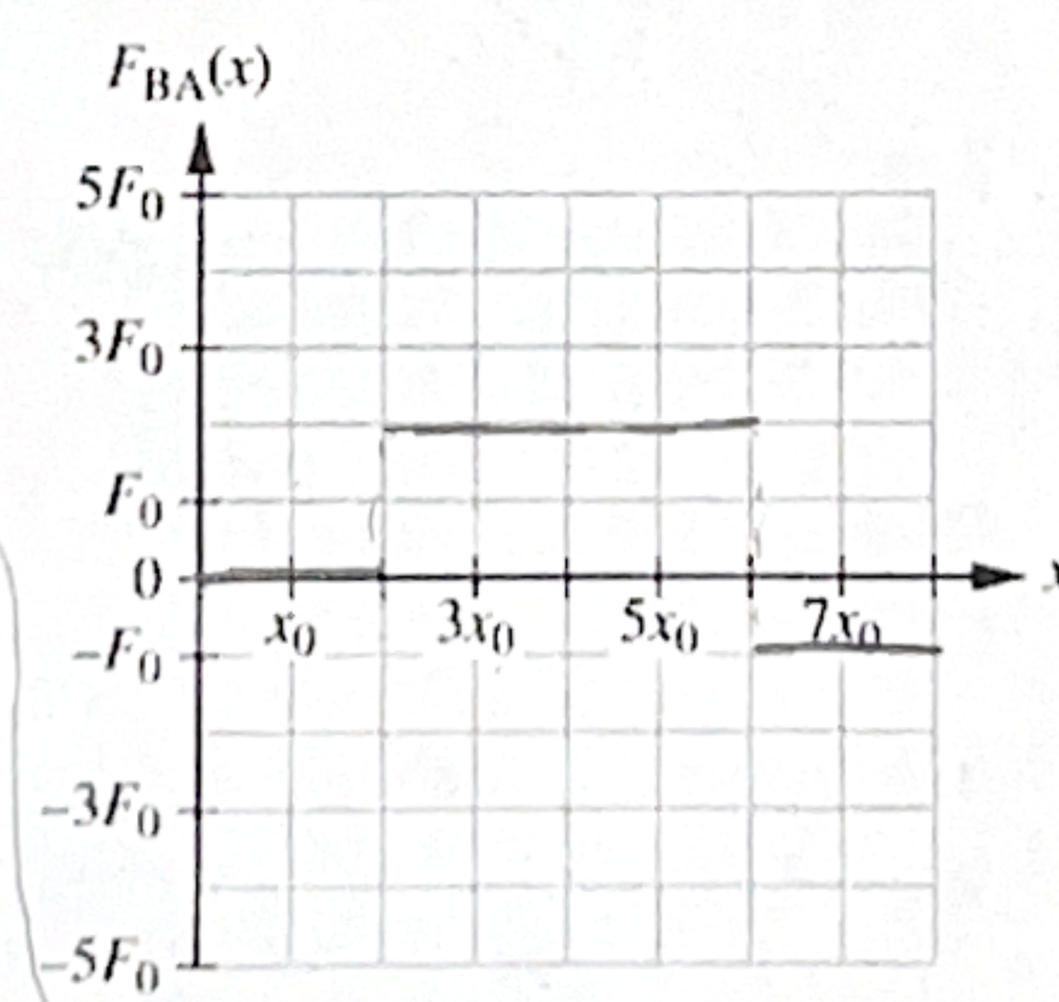


Figure 2

$\Delta U = -\int W$

$\Delta U = -\int_a^b \vec{F} \cdot d\vec{r}$

$\Delta U = -\int_{x_i}^{x_f} -\beta x^2 \hat{i} \cdot dx \hat{i}$

$\Delta U = \int_{x_i}^{x_f} \beta x^2 dx$

$\Delta U = \frac{\beta}{3} x^3 \Big]_{x_i}^{x_f}$

$\Delta U = \frac{\beta}{3} x_f^3 - \frac{\beta}{3} x_i^3$

$U_f - U_i = \frac{\beta}{3} x_f^3 - \frac{\beta}{3} x_i^3$

Let $x_i = -2m$
$x_f = x$

$U(x) - U(2) = \frac{\beta}{3} x^3 - \frac{\beta}{3}(2m)^3$

$U(x) = \frac{\beta}{3} x^3 + \left(\frac{8m^3}{3}\right)\beta$

insert β value

In another scenario, Object A is replaced by Object C. Objects C and B interact with each other through a conservative force $F_{BC}(x) = -\beta x^2$, where $\beta = \frac{3}{8} \frac{\text{kg}}{\text{s}^2\text{m}}$. The potential energy $U_{BC}(x)$ of the Object C-Object B system is defined to be zero when Object C is at position $x = -2$ m. No external forces are exerted on the two object system.

Here is where we evaluate ΔU at some "initial" condition to find $U(x)$

(b) **Derive** an equation for $U_{BC}(x)$. Express your answer only in terms of x.

$U = \left(\frac{3}{8} \frac{kg}{s^2 m}\right)\left(\frac{1}{3} x^3\right) + \frac{8}{3}\left(\frac{3}{8} \frac{kg}{s^2 m}\right) \cdot m^3$

$U = \frac{1}{8}\left(\frac{kg}{s^2 m}\right) x^3 + 1 \frac{kg \cdot m^2}{s^2}$ J

$\Rightarrow \boxed{U(x) = \frac{1}{8}\left(\frac{kg}{s^2 m}\right) x^3 + 1 J}$