*Solutions posted*

Name: ____________

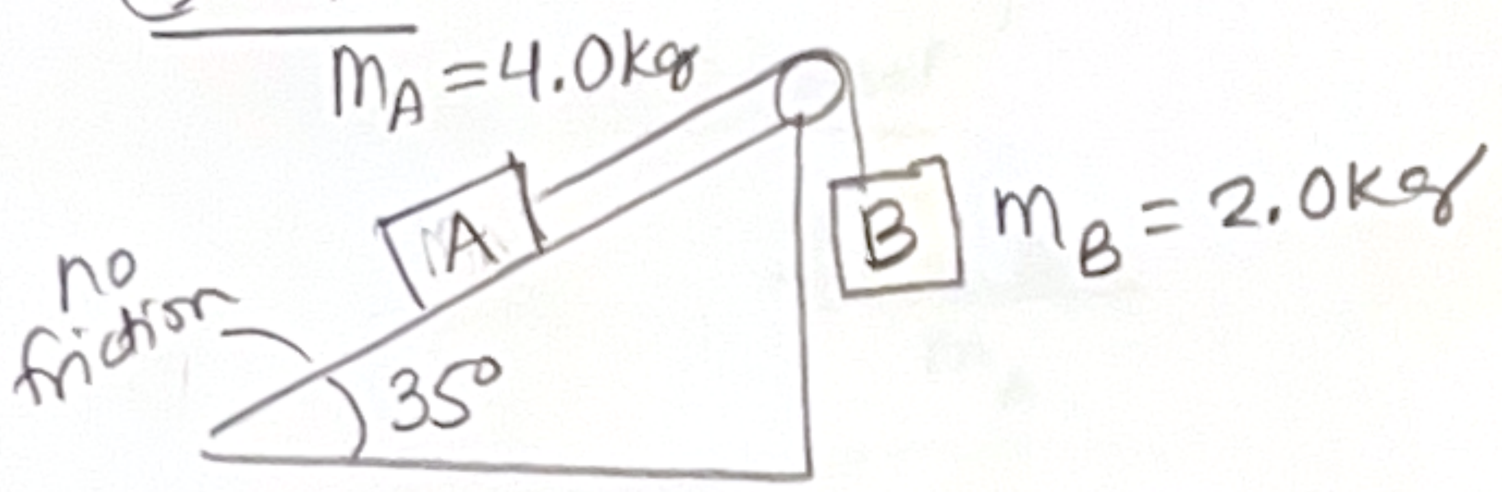
Date: ____________

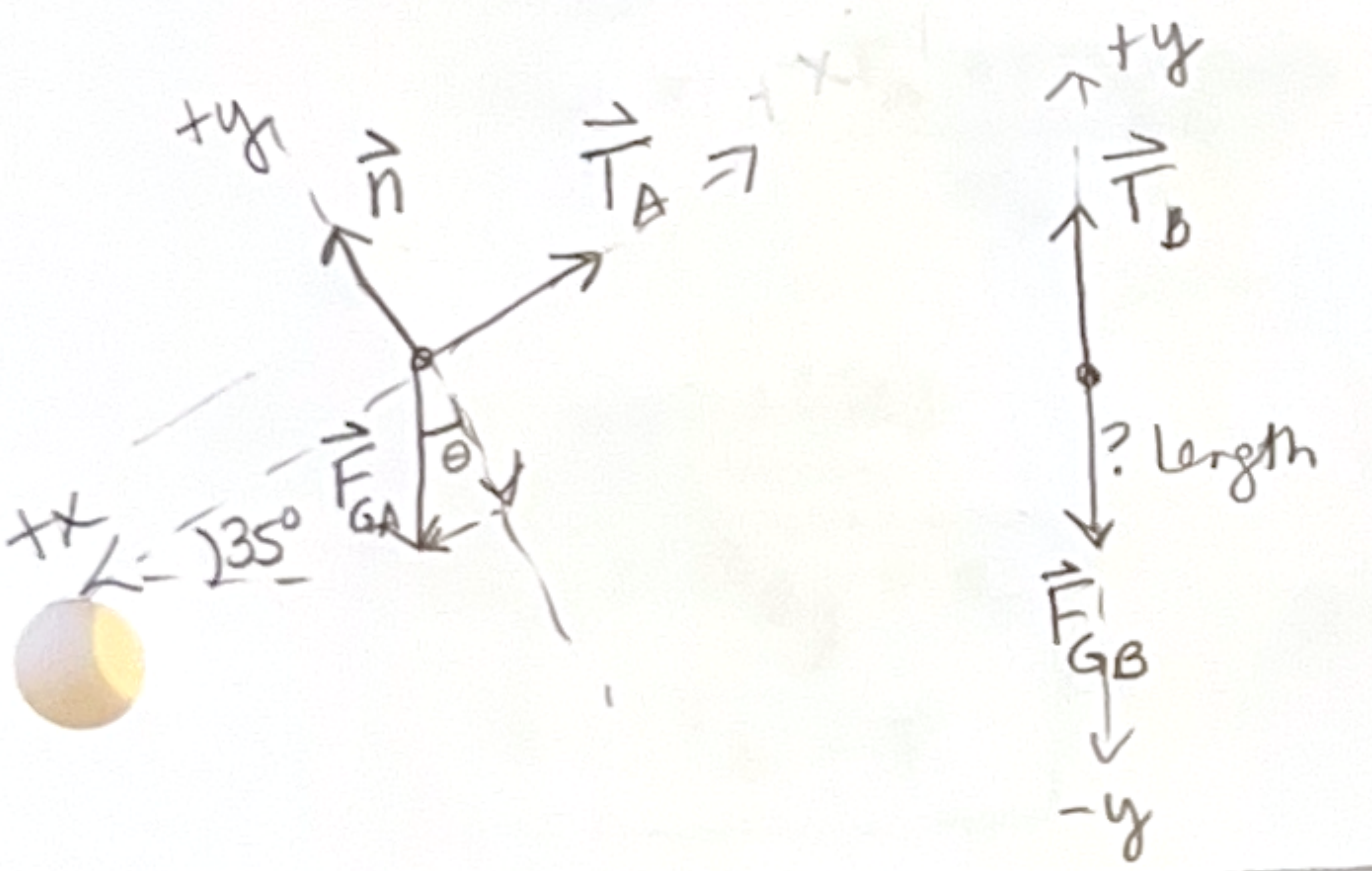
# Two-Body / Calculus

1. p.179 #40. When solving for an expression, show the steps of the General Strategy.

2. p.63 #74. (Hint: First solve the given expression for the velocity so you have a v(t) function to work with.)

① #40




a) If nothing is moving, then we can find $\vec{T}$ using object B, b/c $a_{By}=0$. We can't use object A because we don't know the direction or magnitude of the person's force, so we would have two unknowns: $T_A$ and $F_{person}$.

using object B: (Choose up as +)

$$a_{By} = \frac{\Sigma F_{on\,B\,y}}{m_B}$$

$$0 = \frac{+T_B + (-m_B g)}{m_B}$$

$$0 = T_B - m_B g$$

$$m_B g = T_B$$

$$(2.0kg)(10\tfrac{N}{kg}) = T_B$$

$$\boxed{20N = T_B}$$

b) Which way will it move?

- I need to compare the force trying to pull the blocks "left" to the force trying to pull the blocks "right."

The "left" force is the component of $\vec{F}_{GA}$ parallel to the ramp, $F_{GAx}$:

$$F_{GAx} = m_A g \sin\theta$$

$$F_{GAx} = (4.0kg)(10 N/kg)\sin 35°$$

$$F_{GAx} = 22.9N$$

The "right" force is $F_{GB}$, which is:

$$F_{GB} = m_B g$$

$$F_{GB} = (2.0kg)(10 N/kg)$$

$$F_{GB} = 20N$$

$\boxed{\text{Since } F_{GAx} > F_{GB}, \text{ block A will move down the slope.}}$

- This could also be solved by setting up a coordinate system and using Newton's 2nd law for the two bodies to find the acceleration. If it comes out + or −, you can look at your coordinate system to see which way it is going. I will show this method next:

| For A | | |
|---|---|---|
| $\vec{n}$ | $n_x = 0$ | $n_y = +n$ |
| $\vec{T}_A$ | $T_{Ax} = -T_A$ | $T_{Ay} = 0$ |
| $\vec{F}_{GA}$ | $F_{GAx} = +m_A g \sin\theta$ | $F_{GAy} = -m_A g \cos\theta$ |

| For B | y |
|---|---|
| $\vec{T}_B$ | $T_{By} = +T_B$ |
| $\vec{F}_{GB}$ | $T_{GBy} = -m_B g$ |

Newton's 2nd Law

$$a_{Ax} = \frac{\Sigma F_{on\,A\,x}}{m_A} \qquad a_{Ay} = \frac{\Sigma F_{on\,Ay}}{m_A} \qquad a_{By} = \frac{\Sigma F_{on\,By}}{m_B}$$

$$a_{Ax} = \frac{-T_A + m_A g \sin\theta}{m_A} \qquad 0 = \frac{+n - m_A g \cos\theta}{m_A} \qquad a_{By} = \frac{+T_B + (-m_B g)}{m_B}$$

Common variables

$$\boxed{\begin{array}{l} a_{Ax} = a_{By} = a \\ T_A = T_B = T \end{array}}$$

$$a = \frac{-T + m_A g \sin\theta}{m_A} \qquad a = \frac{T - m_B g}{m_B}$$

Find accel:

$$\underbrace{m_A a = -T + m_A g \sin\theta \qquad m_B a = T - m_B g}_{\text{add:}}$$

$$\begin{aligned} m_A a &= -T + m_A g \sin\theta \\ m_B a &= T - m_B g \\ \hline (m_A + m_B) a &= m_A g \sin\theta - m_B g \end{aligned}$$

$$\boxed{a = \frac{m_A g \sin\theta - m_B g}{(m_A + m_B)}}$$

$$a = \frac{(4.0\,kg)(10\,\tfrac{N}{kg})\sin 35^\circ - (2.0\,kg)(10\,\tfrac{N}{kg})}{(4.0\,kg + 2.0\,kg)}$$

$$a = \frac{22.9\,N - 20\,N}{6\,kg}$$

$$\boxed{a = .48\ m/s^2}$$

This came out with a + accel. Refering back to my coordinate system, I had made the + direction as down the ramp. Therefore this + result means the block A is speeding up down the ramp.

c) Tension in the string

Use either equation that has T. I will choose the one for block B:

$$a = \frac{T - m_B g}{m_B}$$

$$m_B a = T - m_B g$$

$$m_B a + m_B g = T$$

$$\boxed{m_B(a+g) = T}$$

$$(2.0\,kg)(.48\,m/s^2 + 10\,N/kg) = T$$

$$\boxed{21\,N = T}$$

② p.63 #74

solve for $V_x$ first to get $V_x(t)$:

$$V_x^2 = \frac{2P}{m}t$$

$$V_x = \sqrt{\frac{2P}{m}t}$$

$$V_x = \left(\sqrt{\frac{2P}{m}}\right)\left(\sqrt{t}\right)$$

$$V_x = \left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$$

Now I have $V_x(t)$!

a) $a_x = \frac{dV_x}{dt}$

$$a_x = \frac{d}{dt}\left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$$

$$a_x = \sqrt{\frac{2P}{m}}\left(\frac{1}{2}\right)\left(t^{\frac{1}{2}-1}\right)$$

$$a_x = \left(\frac{1}{2}\sqrt{\frac{2P}{m}}\right)t^{-1/2}$$

$$a_x = \frac{1}{2}\sqrt{\frac{2P}{m}} \cdot \frac{1}{\sqrt{t}}$$

$$\boxed{a_x = \frac{1}{2}\sqrt{\frac{2P}{mt}}}$$

b) $\boxed{V_x(t) = \sqrt{\frac{2P}{m}} \cdot \sqrt{t}}$

$$V_x(2s) = \sqrt{\frac{2(3.6\times10^4\,W)}{1200\,kg}} \cdot \sqrt{2}$$

$$V_x(2s) = \boxed{11\,m/s}$$

$$V_x(10s) = \sqrt{\frac{2(3.6\times10^4\,W)}{1200\,kg}} \cdot \sqrt{10}$$

$$V_x(10s) = \boxed{24\,m/s}$$

c) $\boxed{a_x = \frac{1}{2}\sqrt{\frac{2P}{mt}}}$

$$a_x(2s) = \frac{1}{2}\sqrt{\frac{2(3.6\times10^4\,W)}{(1200\,kg)(2s)}}$$

$$a_x(2s) = \boxed{2.7\,m/s^2}$$

$$a_x(10s) = \frac{1}{2}\sqrt{\frac{2(3.6\times10^4\,W)}{(1200\,kg)(10s)}}$$

$$a_x(10s) = \boxed{1.2\,m/s^2}$$