Name: ___________
Date: ___________

Practice: Calculus in Kinematics

Solutions are posted.

1. Write the new physics facts into your booklet.

2. Do **p.60 #33**

3. Do **p.~~60~~ 61 #35** as presented in the text. Then do these parts:
   c. Sketch the *x-t* and v_x-*t* graphs for this time frame: - 1s < t < 5. (Use your calculator or Demos to get the shapes.) On both graphs, mark the points on the graph at the two times you found in part (a). Are they turning points? Explain your reasoning.
   d. You have already know the sign (+ or -) of the acceleration at these two times. Annotate each graph to indicate how the sign of the acceleration can be seen on each graph at each of the times. (That's 4 annotations in total!)

4. A particle's acceleration is described by the function $a_x = (10 - t)$ m/s², where *t* is in *s*. At t = 0 s, its position is *0* m and its velocity is *0* m/s. *(Adapted from p.61 #41)*
   a. Find the particle's velocity as a function of time.
   b. Find the particle's displacement from t = 3 s to t = 10 s.
   c. Find the particle's position as a function of time.
   d. At what time after t = 0 is the velocity again zero? What is the particle's position at that time?

2) p.60 #33

$x = (2t^3 + 2t + 1)$ m

a) position at t=2s?

$x(2s) = 2(2)^3 + 2(4) + 1$

$= \boxed{21 m}$ ← Did you remember units?

b) velocity at t=2s?

$v_x = \frac{dx}{dt} = \frac{d}{dt}(2t^3 + 2t + 1)$

$= 6t^2 + 2$

$v_x(2s) = 6(2)^2 + 2$

$= \boxed{26 \text{ m/s}}$

c) accel at t=2s?

$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(6t^2 + 2) = 12t$

$a_x(2s) = 12(2) = \boxed{24 \text{ m/s}^2}$

3) p.60 #35

35) $x = (2t^3 - 9t^2 + 12)$ m

a) when does $v_x = 0$ m/s?

$v_x = \frac{dx}{dt}$

$v_x = \frac{d}{dt}(2t^3 - 9t^2 + 12)$

$v_x = 6t^2 - 18t$

Set v=0:

$0 = 6t^2 - 18t$

$0 = 6t(t-3)$

$6t = 0$ or $t - 3 = 0$

$\boxed{t = 0s}$ $\boxed{t = 3s}$

b) $a = \frac{dv}{dt} = \frac{d}{dt}(6t^2 - 18t)$

$= 12t - 18$

I need $x(0)$ and $x(3)$, $a(0)$ and $a(3)$

positions: $x = (2t^3 - 9t^2 + 12)\,m$

$x(0) = 0 - 0 + 12 = \boxed{12m}$

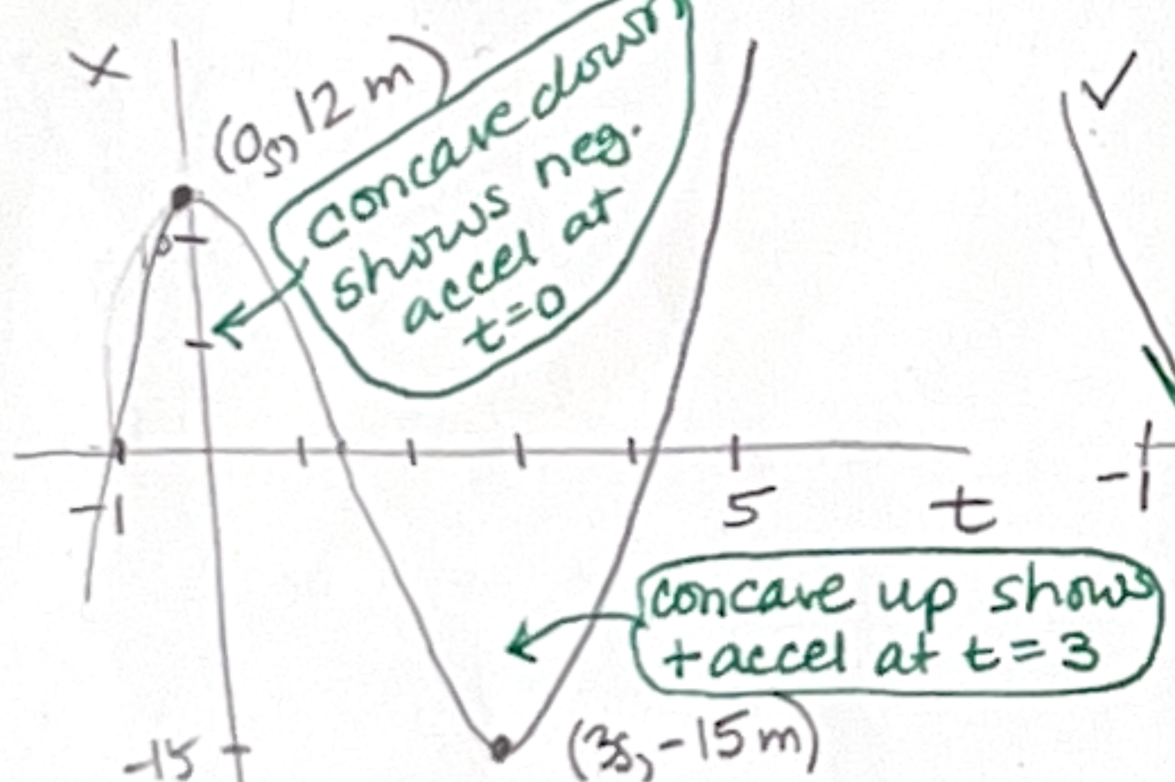
$x(3) = 2(3)^3 - 9(3)^2 + 12 = \boxed{-15m}$

accel: $a = 12t - 18$

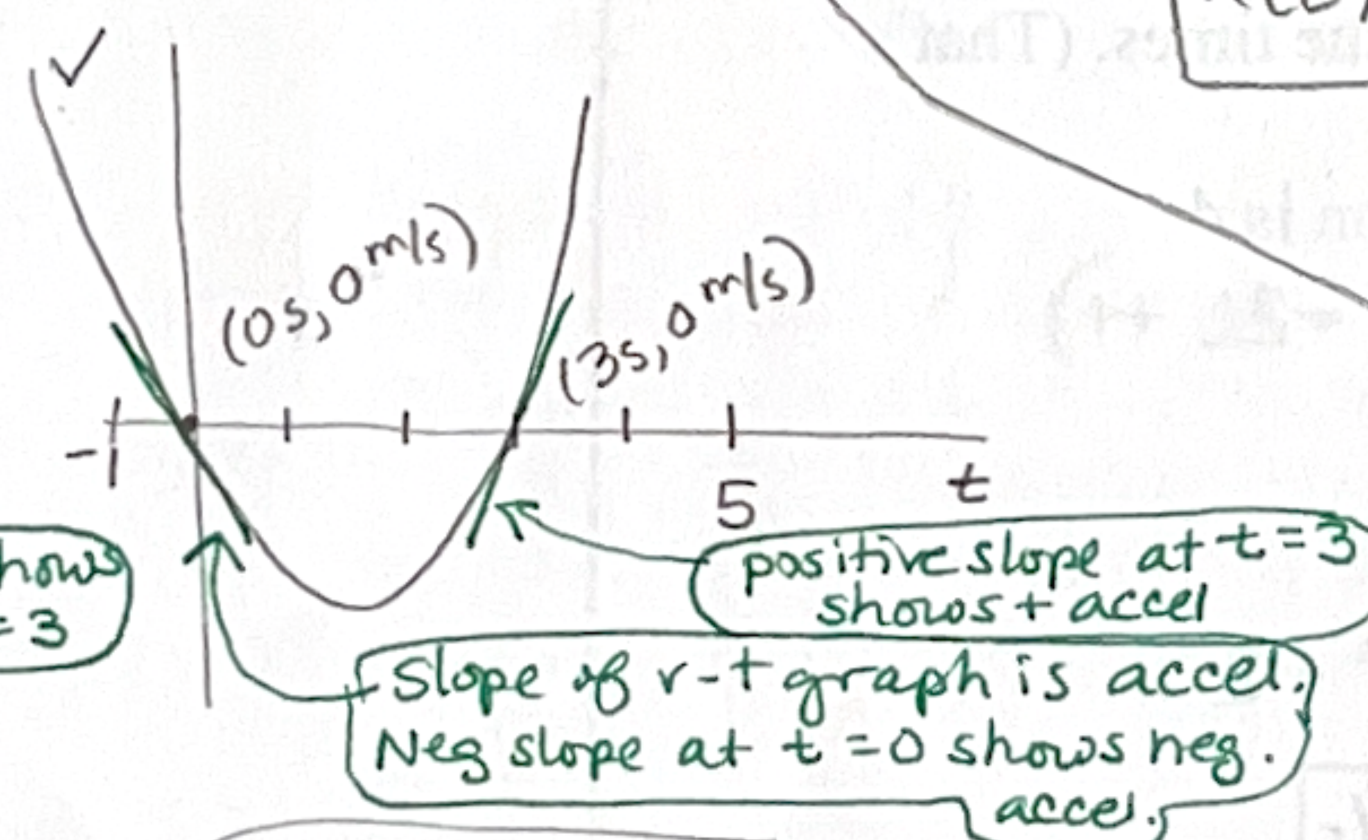
$a(0) = 12(0) - 18 = \boxed{-18\,m/s^2}$

$a(3) = 12(3) - 18 = \boxed{+18\,m/s^2}$

(c),(d)



$x = 2t^3 - 9t^2 + 12$



$v = 6t^2 - 18t$

Yes, they are turning points because the sign of velocity changes at those points. The sign of velocity indicates direction of motion, so if it changes, that means the object changed direction.

④ $a_x = (10 - t)\, m/s^2$

a) Find V_x as a function of time:

$$\Delta V_x = \int_0^t a_x\, dt$$

$$\Delta V_x = \int_0^t (10 - t)\, dt$$

$$V_x(t) - V_x(0) = \left[10t - \tfrac{1}{2}t^2\right]_0^t$$

This is 0, as a given initial condition

$$V_x(t) = (10t - \tfrac{1}{2}t^2) - (0 - 0)$$

$$\boxed{V_x(t) = 10t - \tfrac{1}{2}t^2}$$

b) Find displacement from t=3s to t=10s:

Displacement is area of v-t graph, which is a definite integral: $\Delta x = \int_{t_i}^{t_f} V_x\, dt$

$$\Delta x = \int_3^{10} (10t - \tfrac{1}{2}t^2)\, dt$$

$$\Delta x = \left[10\left(\tfrac{1}{2}t^2\right) - \tfrac{1}{2}\left(\tfrac{1}{3}t^3\right)\right]_3^{10}$$

$$\Delta x = \left[5t^2 - \tfrac{1}{6}t^3\right]_3^{10}$$

$$\Delta x = \left[5(10)^2 - \tfrac{1}{6}(10)^3\right] - \left[5(3)^2 - \tfrac{1}{6}(3)^3\right]$$

$$\Delta x = [500 - 166.7] - [45 - 4.5]$$

$$\boxed{\Delta x = 293\, m}$$

{#4, continued...}

c) Find position as a function of time:

position is the integral of velocity:

$$\Delta x = \int_0^t (10t - \tfrac{1}{2}t^2)\, dt$$

$$x(t) - x(0) = \left[10\left(\tfrac{1}{2}t^2\right) - \tfrac{1}{2}\left(\tfrac{1}{3}t^3\right)\right]_0^t$$

The position at t=0 is 0

$$x(t) = \left[5t^2 - \tfrac{1}{6}t^3\right]_0^t$$

$$x(t) = \left[5t^2 - \tfrac{1}{6}t^3\right] - \left[5(0)^2 - \tfrac{1}{6}(0)^3\right]$$

$$\boxed{x(t) = 5t^2 - \tfrac{1}{6}t^3}$$

d) When does $V_x = 0$?

$$V_x(t) = 10t - \tfrac{1}{2}t^2 = 0$$

$$t(10 - \tfrac{1}{2}t) = 0$$

$t = 0$

$10 - \tfrac{1}{2}t = 0$

$10 = \tfrac{1}{2}t$

$\boxed{20s = t}$

position at t=20s?

$$x(t) = 5t^2 - \tfrac{1}{6}t^3$$

$$x(20s) = 5(20s)^2 - \tfrac{1}{6}(20s)^3$$

$$= \boxed{667\, m}$$