Solutions posted

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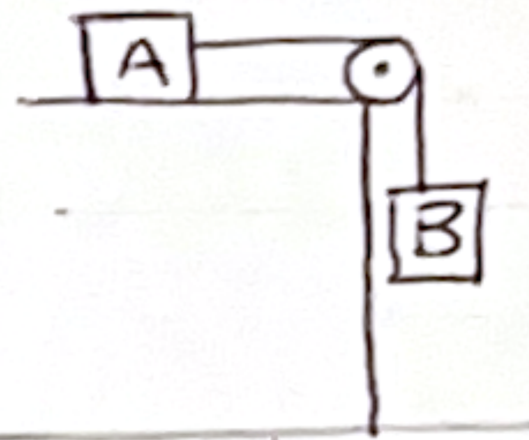
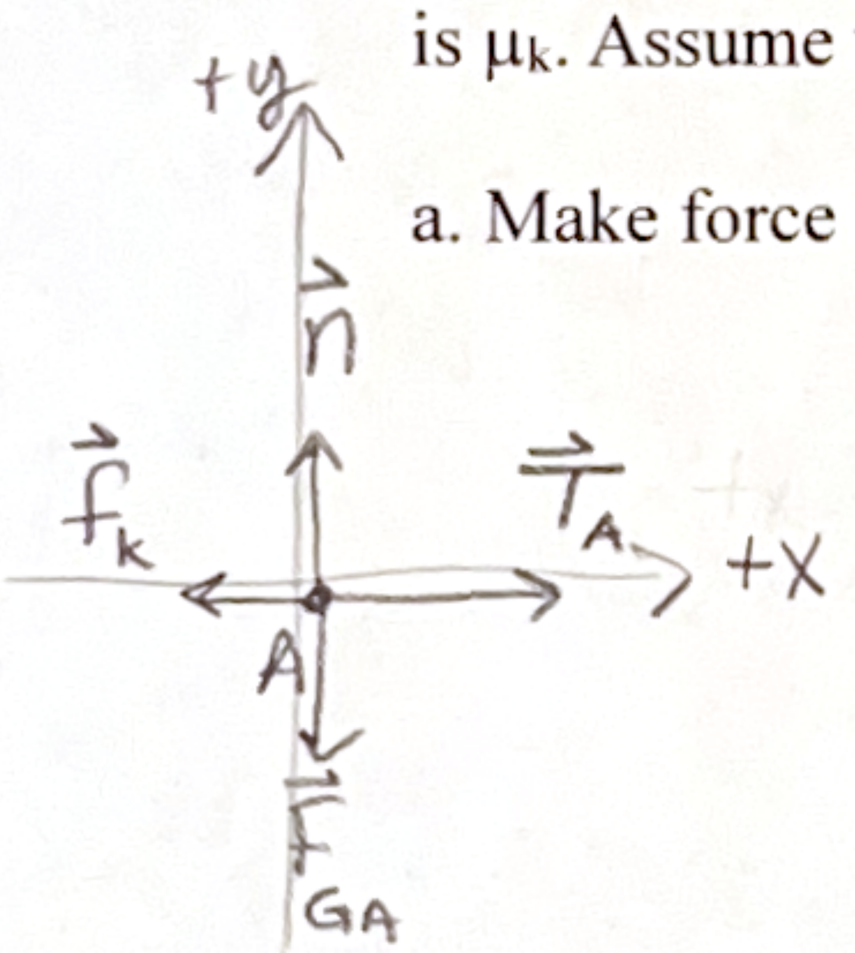
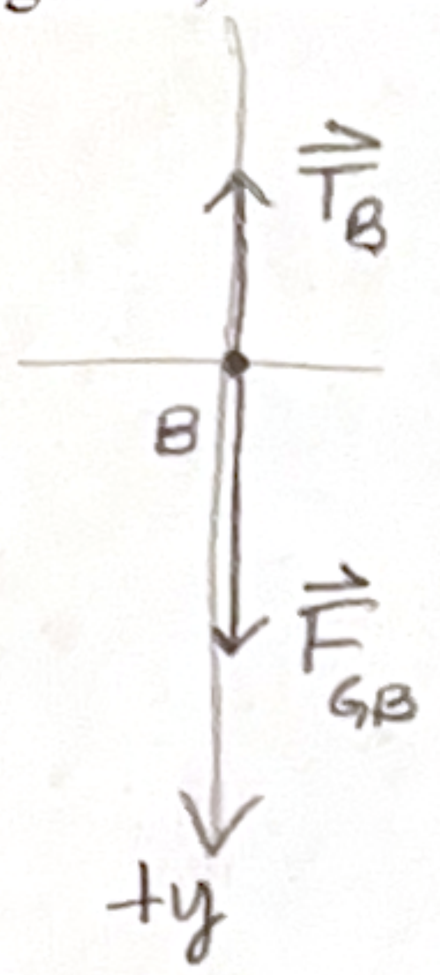
Modified Atwood with Friction

1. Block A of mass m_A is on a table, and it is attached to Block B of mass m_B by a string that passes over an ideal pulley. The coefficient of kinetic friction on the table is μ_k. Assume the system is accelerating.



a. Make force diagrams, coordinate systems, and force organizers





For A	x	y
$\vec{T}_A$	$T_{Ax}=+T_A$	$T_{Ay}=0$
$\vec{f}_k$	$f_{kx}=-\mu_k n$	$f_{ky}=0$
$\vec{n}$	$n_x=0$	$n_y=+n$
$\vec{F}_{GA}$	$F_{GAx}=0$	$F_{GAy}=-m_A g$

	x	y
$\vec{T}_B$	$T_{Bx}=0$	$T_{By}=-T_B$
$\vec{F}_{GB}$	$F_{GBx}=0$	$F_{GBy}=+m_B g$

b. Write Newton's second law for each object, in every direction that has forces.

$$a_{Ax}=\frac{\Sigma F_{on\,Ax}}{m_A} \qquad a_{Ay}=\frac{\Sigma F_{on\,Ay}}{m_A} \qquad a_{By}=\frac{\Sigma F_{on\,By}}{m_B}$$

$$a_{Ax}=\frac{+T_A+(-\mu_k n)}{m_A} \qquad 0=\frac{+n+(-m_A g)}{m_A} \qquad a_{By}=\frac{+m_B g+(-T_B)}{m_B}$$

c. Define common variables and rewrite your equations.

$$\boxed{\begin{array}{l} a_{Ax}=a_{By}=a \\ T_A=T_B=T \end{array}}$$

$$a=\frac{T-\mu_k n}{m_A} \qquad 0=n-m_A g \qquad a=\frac{m_B g-T}{m_B}$$

d. Find the acceleration of block A in terms of given variables and fundamental constants.

eliminate fractions:

$$m_A a=T-\mu_k n \quad ① \qquad n=m_A g \quad ② \qquad m_B a=m_B g-T \quad ③$$

now add 1st + 3rd equations:

$$\begin{array}{rl} & m_A a=T-\mu_k n \quad ① \\ + & m_B a=m_B g-T \quad ③ \\ \hline & a(m_A+m_B)=m_B g-\mu_k n \end{array}$$

solve for a: $a=\dfrac{m_B g-\mu_k n}{(m_A+m_B)}$ oops! I need to replace n.

$$\boxed{a=\frac{m_B g-\mu_k(m_A g)}{(m_A+m_B)}}$$

e. Find the tension in the string in terms of given variables and fundamental constants.

Substitute the acceleration into any equation in (d) that has a T:

using Eqn ①: $m_A a=T-\mu_k n$

$T=m_A a+\mu_k n$

$$T=m_A\left[\frac{m_B g-\mu_k m_A g}{(m_A+m_B)}\right]+\mu_k m_A g$$

using Eqn ③: $T=m_B g-m_B a$

$$T=m_B g-m_B\left[\frac{m_B g-\mu_k m_A g}{(m_A+m_B)}\right]$$

Both simplify to this:

$$T=\frac{m_A m_B g(1+\mu_k)}{m_A+m_B}$$

see p.3 for proof

2. If the acceleration of a block is given by $a_x = 3t - 4t^2$, where distance is in meters and time is in seconds, find the block's change in velocity from t = 1s to t = 3s.

$$\Delta V_x = \int_{t_i}^{t_f} a_x \, dt$$

$$\Delta V_x = \int_{1}^{3} (3t - 4t^2) \, dt$$

$$\Delta V_x = \left[3\left(\frac{1}{2}t^2\right) - 4\left(\frac{1}{3}t^3\right) \right]_1^3$$

$$\Delta V_x = \left[\frac{3}{2}t^2 - \frac{4}{3}t^3 \right]_1^3$$

$$\Delta V_x = \left[\frac{3}{2}(3)^2 - 4(3)^3 \right] - \left[\frac{3}{2}(1)^2 - \frac{4}{3}(1)^3 \right]$$

$$\Delta V_x = \frac{27}{2} - 108 - \frac{3}{2} + \frac{4}{3}$$

$$\boxed{\Delta V_x = -94.7 \, m/s}$$

Showing that tension T comes out the same regardless of equation used:

using ①:

$$m_A a = T - \mu_K n$$

$$m_A a + \mu_K n = T$$

$$T = m_A a + \mu_K (m_A g)$$

now substitute for a:

$$T = m_A \left[\frac{m_B g - \mu_K m_A g}{m_A + m_B}\right] + \mu_K m_A g$$

$$T = \frac{m_A m_B g - \mu_K m_A^2 g}{m_A + m_B} + \mu_K m_A g \left(\frac{m_A + m_B}{m_A + m_B}\right)$$

to get a common denominator

$$T = \frac{m_A m_B g - \cancel{\mu_K m_A^2 g} + \cancel{\mu_K m_A^2 g} + \mu_K m_A m_B g}{m_A + m_B}$$

$$T = \frac{m_A m_B g + \mu_K m_A m_B g}{m_A + m_B}$$

$$\boxed{T = \frac{m_A m_B g (1 + \mu_K)}{(m_A + m_B)}}$$

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using ③:

$$m_B a = m_B g - T$$

$$T = m_B g - m_B a$$

substitute a: $T = m_B g - m_B \left[\frac{m_B g - \mu_K m_A g}{m_A + m_B}\right]$

$$T = m_B g - \left(\frac{m_B^2 g - \mu_K m_A m_B g}{m_A + m_B}\right)$$

get common denominator & distribute minus sign: $T = m_B g \left(\frac{m_A + m_B}{m_A + m_B}\right) + \left(\frac{-m_B^2 g + \mu_K m_A m_B g}{m_A + m_B}\right)$

$$T = \frac{m_A m_B g + \cancel{m_B^2 g} - \cancel{m_B^2 g} + \mu_K m_A m_B g}{m_A + m_B}$$

$$T = \frac{m_A m_B g + \mu_K m_A m_B g}{m_A + m_B}$$

$$\boxed{T = \frac{m_A m_B g (1 + \mu_K)}{(m_A + m_B)}}$$