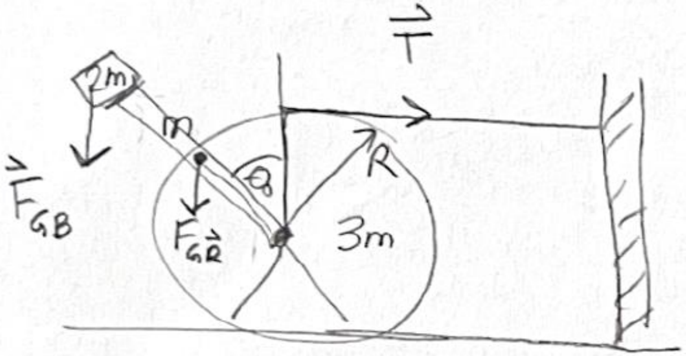
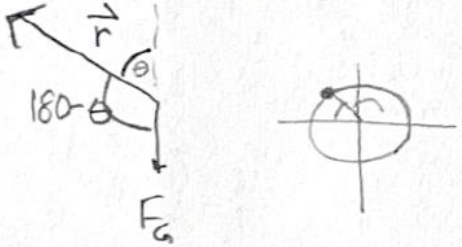
FRQ-6 Scoring Guide

Part a (5 points)



a) $\Sigma\tau = 0$

1 point: For indicating that the net torque is zero, or that the clockwise and counterclockwise torques are equal



$\sin(180-\theta_0) = \sin\theta_0$

1 point: For a correct expression for the torque exerted by the string

1 point: For a correct expression for the torque exerted by the block

1 point: For a correct expression for the torque exerted by the rod

$-RT + F_{GB}(2R)\sin(180-\theta_0) + F_{GR}(R)\sin(180-\theta_0) = 0$

$-RT + \boxed{2mg(2R)\sin\theta_0} + \boxed{mRg\sin\theta_0} = 0$

$4mg\cancel{R}\sin\theta_0 + mg\cancel{R}\sin\theta_0 = \cancel{R}T$

$\boxed{5mg\sin\theta_0} = T$

1 point: For adding the counterclockwise torques and setting the sum equal to the clockwise torque (this point not awarded for just one torque)

- Only four points could be earned if the wrong trigonometric function was used.
- Only three points could be earned if no trigonometric function was used.

Part b(i) (4 points)

$I = I_{disk} + I_{rod} + I_{block}$

1 point: For indicating that the rotational inertia is the sum of the inertias of the disk, rod, and block.

b) immediately:

i) $\Sigma\tau = I\alpha$

$4mgR\sin\theta + mgR\sin\theta_0 = I\alpha$

$\boxed{5mgR\sin\theta_0} = \left(\frac{65}{6}\right)mR^2\alpha$

1 point: For a value of torque equal to the tension in (a) times R

$5\left(\frac{6}{65}\right)\frac{g\sin\theta_0}{R} = \alpha$

$\frac{30}{65}\frac{g\sin\theta_0}{R} = \alpha$

$\boxed{\frac{6}{13}\frac{g\sin\theta_0}{R} = \alpha}$

1 point: For an answer consistent with the values use for torque and rotational inertia

$I = \frac{3}{2}mR^2 + \frac{4}{3}mR^2 + (2m)(2R)^2$

$= \frac{3}{2}mR^2 + \frac{4}{3}mR^2 + 8mR^2$

$= \frac{9}{6}mR^2 + \frac{8}{6}mR^2 + \frac{48}{6}mR^2$

$= \frac{65}{6}mR^2$

1 point: For calculating the total rotational inertia

Part b(ii) (1 points)

bii) linear accel of the end of the rod:

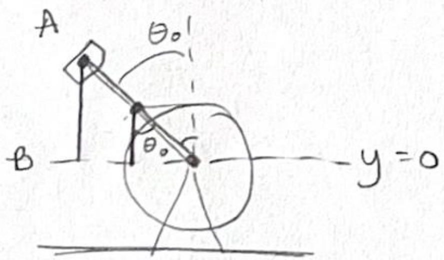
$$a_t = r\alpha \quad , \text{ where } r = 2R$$

$$= (2R)\left(\frac{6}{13}\frac{g\sin\theta_0}{R}\right)$$

$$= \boxed{\frac{12}{13} g\sin\theta_0}$$

1 point: For substituting the value of α and the correct radius, 2R.

c) linear speed of the mass at the end of the rod when it is horizontal



Part c (5 points)

Initial height of center of mass of rod: $h_r = R\cos\theta_0$

Initial height of mass at end of rod: $h_m = (2R)\cos\theta_0$

1 point: For indicating that energy is conserved

$$E_A + W_{ext} = E_B$$

$$(U_G)_{rod} + (U_G)_{mass} + 0 = \frac{1}{2}I\omega^2$$

1 point: For indicating the correct kinetic energy when the rod is horizontal

1 point: For indicating that the potential energy of two bodies (the rod and the block) changes

$$mg(R\cos\theta_0) + 2mg(2R\cos\theta_0) = \frac{1}{2}\left(\frac{65}{6}mR^2\right)\omega^2$$

$$5\cancel{m}g\cancel{R}\cos\theta_0 = \frac{65}{12}\cancel{m}R^{\cancel{2}}\omega^2$$

$$\sqrt{\frac{12g\cos\theta_0}{13R}} = \omega$$

1 point: For the correct expressions for these two potential energies

Now I need the linear (tangential) speed of the mass:

$$v_t = r\omega$$

1 point: For using the relationship between linear and angular speed, and substituting ω and the correct radius of 2R

$$= (2R)\left(\sqrt{\frac{12g\cos\theta_0}{13R}}\right)$$

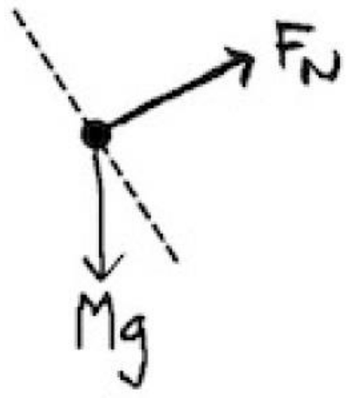
$$= 4R\sqrt{\frac{3g\cos\theta_0}{13R}}$$

$$= \boxed{4\sqrt{\frac{3gR\cos\theta_0}{13}}}$$

FRQ-7 Scoring Guide

Part a (2 points)

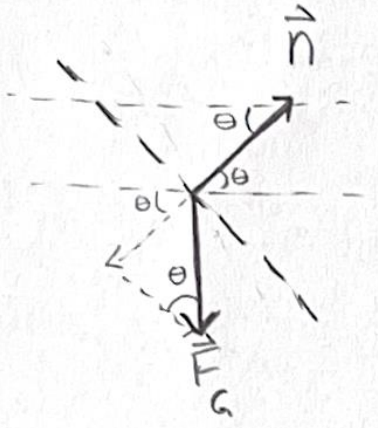
1 point: For either a weight force or a normal force, correctly drawn and labeled



1 point: For a second correct force and no additional forces, arrows, or components

Part b (1 point)

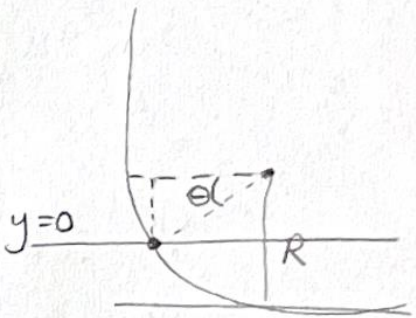
b)



$$\Sigma F_r = n - F_{Gr}$$
$$= \boxed{n - F_G \sin\theta}$$

Alternate solution

① use energy to find v_c



$$E_A = E_C$$
$$Mg\left(R\sin\theta + \frac{3R}{4}\right) = \frac{1}{2}Mv_c^2$$
$$2g\left(R\sin\theta + \frac{3R}{4}\right) = v_c^2$$

② Then, $\Sigma F_r = ma_r$

$$\Sigma F_r = M\left(\frac{v_c^2}{R}\right)$$
$$\Sigma F_r = \frac{M\left(2g\left(R\sin\theta + \frac{3R}{4}\right)\right)}{R}$$
$$\Sigma F_r = \boxed{2Mg\left(\sin\theta + \frac{3}{4}\right)}$$

1 point: For a correct expression for the centripetal force in terms of the forces drawn in part a.

OR, for applying conservation of energy and obtaining this correct answer.

Part c (2 points)

c) Energy principle from A to D:

$E_A + W_{ext} = E_D$

• $W_{ext} = 0$ if the compartment and earth are in the system.

• Initial height above D is $R + \frac{3R}{4} = \frac{7}{4}R$

$\cancel{m}gh_A + 0 = \frac{1}{2}\cancel{m}v_D^2$

$g\left(\frac{7R}{4}\right) = \frac{1}{2}v_D^2$

$\boxed{\sqrt{\frac{7gR}{2}} = v_D}$

1 point: 1 point: For applying conservation of energy, with the loss of potential energy equal to the kinetic energy at point D

1 point: For a correct answer

Part d (3 points)

d) Energy principle:

$$E_D + W_{ext} = E_E$$

1 point: *For equating the kinetic energy of the compartment at point D to the thermal energy at E*

$$\frac{1}{2}Mv_D^2 + 0 = E_{th}$$

$$\frac{1}{2}M\left(\sqrt{\frac{7gR}{2}}\right)^2 = f_k \Delta s$$

$$\frac{1}{2}M\left(\frac{7gR}{2}\right) = \mu_k Mg(3R)$$

1 point: *For substituting the expression for v_D from part c., and d = 3R*

1 point: *For a correct expression for the frictional force*

$$\frac{7}{4}MgR = \mu_k Mg(3R)$$

$$\frac{7}{12} = \mu_k$$

$$\boxed{.58 = \mu_k}$$

Newton's Laws + Kinematics:

$$\Sigma F_x = ma_x$$

1 point: *For using both N2L and a kinematics equation*

$$-f_k n = Ma_x$$

$$-\mu_k(Mg) = Ma_x$$

1 point: *For a correct expression for the frictional force*

$$-\mu g = a_x$$

1 point: *For substituting the expression for v_D from part c., and d = 3R*

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$

$$0 = \left(\sqrt{\frac{7gR}{2}}\right)^2 + 2(-\mu g)(3R)$$

$$0 = \frac{7gR}{2} - 6\mu_k gR$$

$$-\frac{7gR}{2} = -6\mu_k gR$$

$$\frac{7}{12} = \mu_k$$

$$\boxed{.58 = \mu_k}$$

Part e(i) (2 points)

e) i) Differential equation for $v(t)$

$$\Sigma F_x = ma_x$$

$$\boxed{-kv = m\frac{dv}{dt}}$$

1 point: For substituting the time derivative of velocity for the acceleration

1 point: For substituting the braking force into Newton's second law as the net force

Part e(ii) (2 points)

ii) Solve the differential equation

$$-kv = m\frac{dv}{dt}$$

$$-\frac{k}{m}dt = \frac{dv}{v}$$

$$\int -\frac{k}{m}dt = \int \frac{dv}{v}$$

$$-\frac{k}{m}t = \ln v + C$$

1 point: For separating the variables and integrating

evaluate C.

At $t=0, v = v_D$

$$-\frac{k}{m}(0) = \ln v_D + C$$

$$-\ln v_D = C$$

write expression with the constant value:

$$-\frac{k}{m}t = \ln v - \ln v_D$$

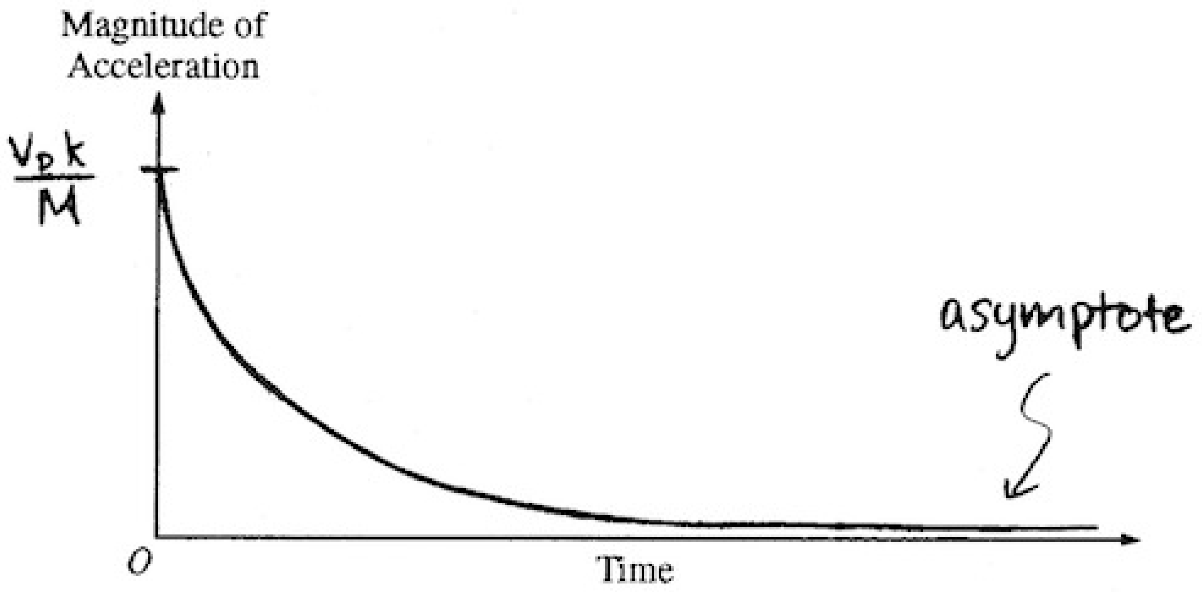
$$-\frac{k}{m}t = \ln\left(\frac{v}{v_D}\right)$$

$$e^{-\frac{k}{m}t} = \frac{v}{v_D}$$

$$\boxed{v_D e^{-\frac{k}{m}t} = v}$$

1 point: For a correct expression for the velocity as a function of time

Part e(iii) (3) points)



1 point: For a graph with a finite intercept on the vertical axis

1 point: For a graph that is concave upward and asymptotic to zero

1 point: For labeling the initial acceleration with the correct value