Using Rotational Quantities

1. $60°$

$$60°\left(\frac{2\pi \text{ rad}}{360°}\right) = \boxed{1.0 \text{ rad}} = \boxed{\frac{\pi}{3} \text{ rad}}$$

$$60°\left(\frac{1 \text{ rev}}{360°}\right) = \boxed{\frac{1}{6} \text{ rev}} = \boxed{.17 \text{ rev}}$$

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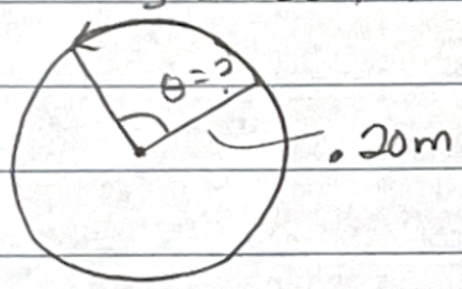
2. $s = 50\text{cm}$



$s = \theta r$

$.50\text{m} = \theta(.20\text{m})$

$\boxed{2.5 \text{ rad} = \theta}$

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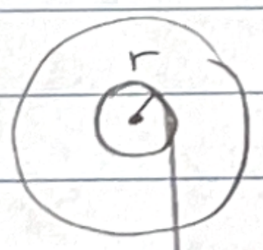
3. 

$r = .50\text{cm}$

$\theta = 30.0 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$

$= 60\pi \text{ rad}$

The length of string is the same as the arc length the axle turns through.

$s = \theta r$

$= (60\pi)(.50 \text{ cm})$

$= \boxed{94.2 \text{ cm}}$

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4. $\theta = 40.0 \text{ rev}$

$\Delta t = 1.00 \text{ min}$

$\omega = ?$ in rev/s, rad/s

$$\omega = \frac{40 \text{ rev}}{1.0 \text{ min}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$= \boxed{0.67 \text{ rev/s}}$$

$$\omega = \frac{40 \text{ rev}}{1.0 \text{ min}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = \boxed{4.2 \text{ rad/s}}$$

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5. $\Delta t = 5.0\text{s}$

$\omega_i = 5.0 \times 10^2 \text{ rad/s}$

$\omega_f = 3.2 \times 10^2 \text{ rad/s}$

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{3.2 \times 10^2 \text{ rad/s} - 5.0 \times 10^2 \text{ rad/s}}{5.0\text{s}}$$

$$= \boxed{-36 \text{ rad/s}^2}$$

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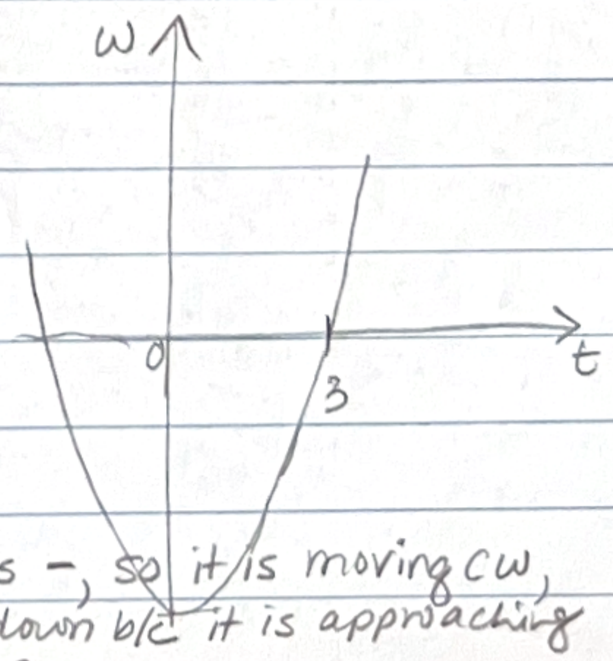
6. $\theta = t^3 - 27.0t + 4.00$

a) $\omega = \frac{d\theta}{dt}$

$\boxed{\omega = 3t^2 - 27}$

b) $\alpha = \frac{d\omega}{dt}$

$\boxed{\alpha = 6t}$



- at $t = 3$ it is instantaneously at rest
- $t > 3$, it is moving ccw because ω is + and speeding up

c) $0 < t < 3$, ω is −, so it is moving cw, and slowing down b/c it is approaching $\omega = 0$ at $t = 3$.

7) $\alpha = 5.00t^3 - 4.00t$

at $t=0$, $\omega = 5.00\ rad/s$

$\theta = 2.00\ rad$

a) Find $\omega(t)$

$\alpha = 5.00t^3 - 4.00t$

$\frac{d\omega}{dt} = 5.00t^3 - 4.00t$

$\int d\omega = \int (5.00t^3 - 4.00t)\,dt$

$\omega = \frac{5}{4}t^4 - \frac{4t^2}{2} + C$

at $t=0$,

$5.00\ rad/s = \frac{5}{4}(0) - 2(0) + C$

$5.00\ rad/s = C$

$\therefore \boxed{\omega = \frac{5}{4}t^4 - 2t^2 + 5}$

b) $\omega = \frac{d\theta}{dt}$

So, $\frac{d\theta}{dt} = \frac{5}{4}t^4 - 2t^2 + 5$

$\int d\theta = \int \left(\frac{5}{4}t^4 - 2t^2 + 5\right) dt$

$\theta = \frac{5}{4}\left(\frac{1}{5}\right)t^5 - \frac{2}{3}t^3 + 5t + C$

$\theta = \frac{t^5}{4} - \frac{2}{3}t^3 + 5t + C$

at $t=0$,

$2.00\ rad = \frac{(0)^5}{4} - \frac{2}{3}(0)^3 + 5(0) + C$

$2.00\ rad = C$

$\therefore \boxed{\theta = \frac{t^5}{4} - \frac{2}{3}t^3 + 5t + 2}$

8) $\omega_i = 33\frac{1}{3}\ rev/min$

$\omega_f = 0$

$\Delta t = 30.0s = (0.50\ min)$

$\alpha = ?$

$\theta = ?$

a) $\omega_f = \omega_i + \alpha \Delta t$

$0 = (33.333\ \frac{rev}{min}) + \alpha(.50\ min)$

$\boxed{\alpha = -66.7\ rev/min^2}$

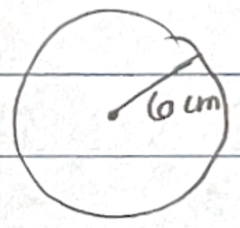
b) $\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2$

$= (33.333\ \frac{rev}{min})(0.50\ min) + \frac{1}{2}(-66.7\ \frac{rev}{min^2})\cdot (.5min)^2$

$= 16.67\ rev - 8.34\ rev$

$= \boxed{8.3\ rev}$

9)



For cord to unwind,
Length of cord = arc length that passes a point at the edge.

$s = 3.00m$

$r = 6\ cm$

$\omega_i = 0$

$\alpha = 2.00\ rad/s^2$

$s = r\theta$

$3.00\ m = (.06\ m)\theta$

$\boxed{50\ rad = \theta}$

10) $\omega = 4.00\ rad/s$ for 10.0 s

Then $\alpha = ?$ for 50.0 s, and

$\omega_f = 52.0\ rad/s$.

$\Delta\theta = ?$ for the whole time period

→ First 10 seconds:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$4.00\ rad/s = \frac{\Delta\theta}{10.0 s}$$

$$40\ rad = \Delta\theta$$

→ Next 50 s:

$\omega_i = 4.00\ rad/s$

$\omega_f = 52.0\ rad/s$

$\Delta t = 50.0 s$

$\Delta\theta = ?$

$$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$$

$$\Delta\theta = \frac{1}{2}\left(4.00\frac{rad}{s} + 52.0\frac{rad}{s}\right)(50.0 s)$$

$$\Delta\theta = 1400\ rad$$

→ For the whole time interval of 60 s,

$\Delta\theta = 40\ rad + 1400\ rad$

$\boxed{\Delta\theta = 1440\ rad}$