Unit 9 Practice

1. A 6.0-cm-diameter gear rotates with angular velocity $\omega = t^3 + t + 1$, where ω is in rad/s and time is in seconds. At $t = 0$, the disk had an angular position of $\theta = 3.0$ rad.

a) Find the angular position of the disk as a function of time.
b) Find the angular acceleration of the disk as a function of time.
c) Find the angular acceleration of the disk at $t = 2.0$ seconds.

a) $\omega = \frac{d\theta}{dt}$

$\int d\theta = \int \omega \, dt$

$\theta = \int (t^3 + t + 1)\,dt$

$\theta = \frac{t^4}{4} + \frac{t^2}{2} + t + C$

at $t=0$, $\theta = 3$ rad, so

$3\text{ rad} = 0 + 0 + 0 + C$

$3\text{ rad} = C$

$\therefore \boxed{\theta(t) = \frac{t^4}{4} + \frac{t^2}{2} + t + 3}$

b) $\alpha = \frac{d\omega}{dt}$

$\alpha = \frac{d}{dt}(t^3 + t + 1)$

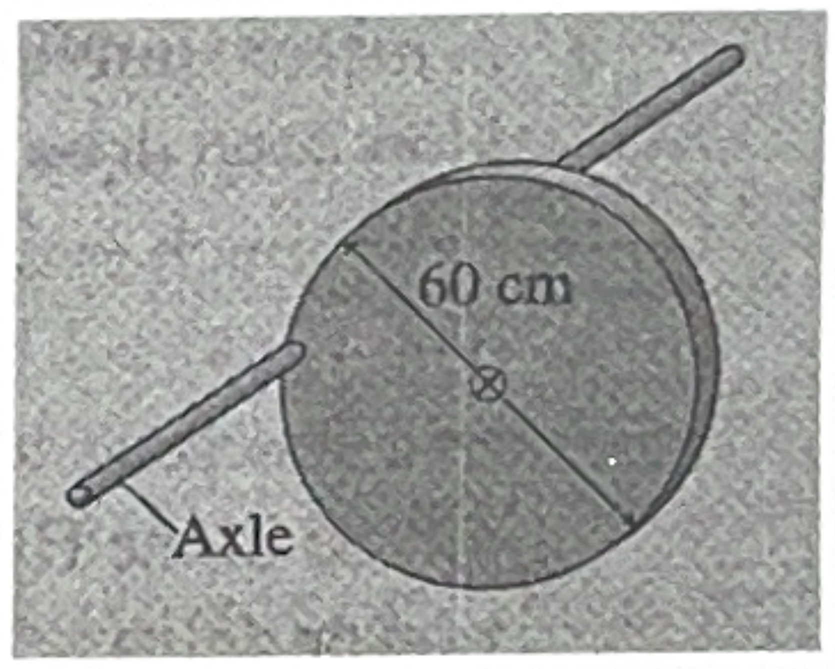
$\boxed{\alpha = 3t^2 + 1}$

c) α at $t = 2.0$ s:

$\alpha = 3(2s)^2 + 1$

$\boxed{\alpha = 13 \text{ rad/s}^2}$

2. A 5.0 kg, 60-cm diameter disk in the figure rotates on an axle passing through one edge. The axle is parallel to the floor. The rotational inertia for a disk rotating around an axle passing though one edge is given by $I = \frac{3}{2}MR^2$. The cylinder is held with the center of mass at the same height as the axle, then released. (*Adapted from p.334 #72*)



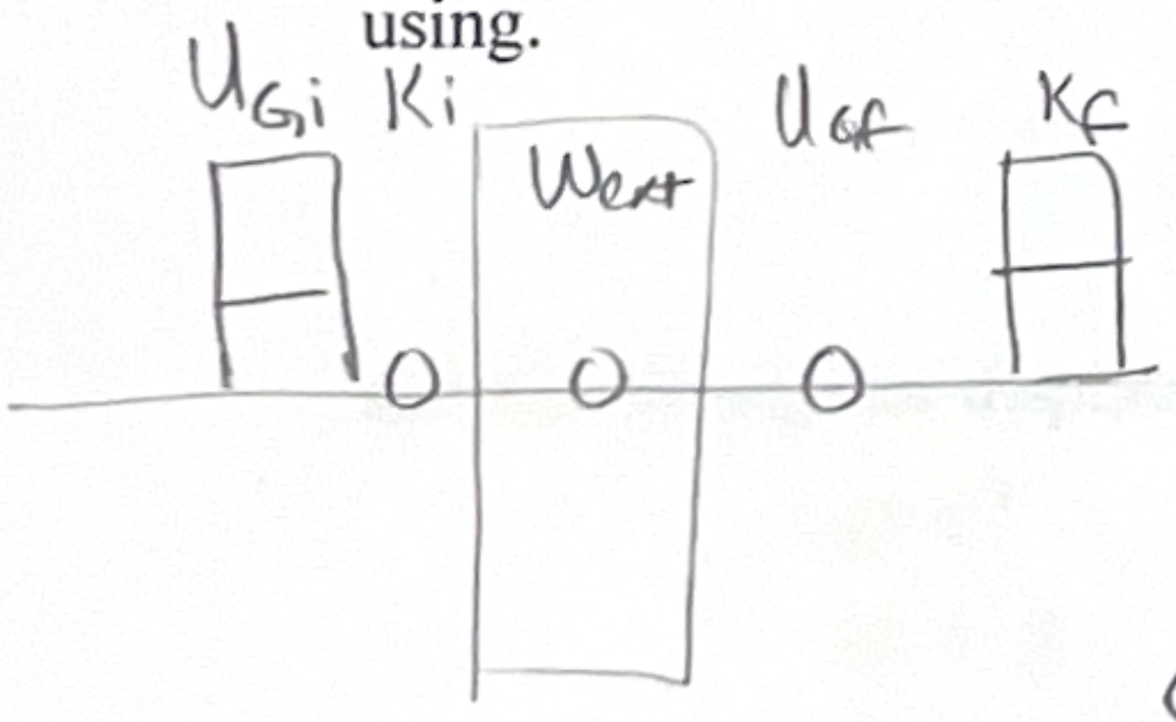
a) Circle the option in each row that best describes this process:

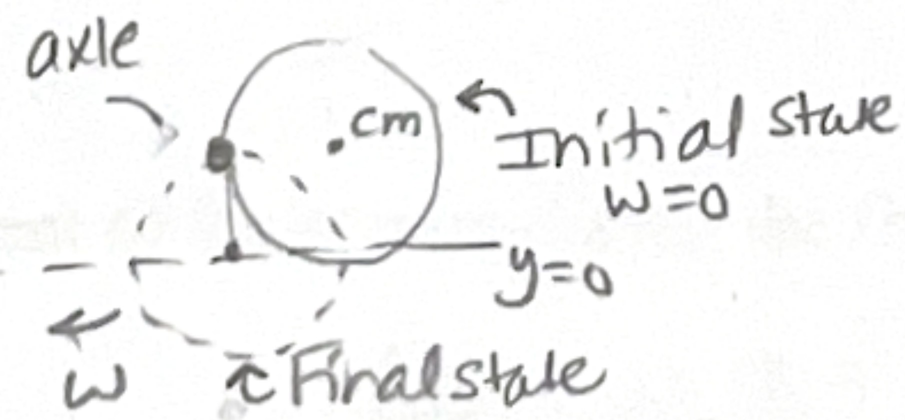
Constant net torque	Changing net torque
Constant angular acceleration	Changing angular acceleration

Energy is the best approach!

At the instant when the cylinder is directly below the axle,

b) find the cylinder's angular velocity. Include the appropriate diagram for the method you are using.





Initial height of C.M. is R

System: disk, earth
(external force from axle, but it does no work)

$E_i + W_{ext} = E_f$

$U_{Gi} + 0 = K_f$

$MgR = \frac{1}{2} I \omega_f^2$

$MgR = \frac{1}{2}\left(\frac{3}{2}MR^2\right)\omega_f^2$

$gR = \frac{3}{4}R^2\omega_f^2$

$\omega_f = \sqrt{\frac{4g}{3R}} = \sqrt{\frac{4(9.8 N/kg)}{3(.3m)}} = \boxed{6.6 \frac{rad}{s}}$

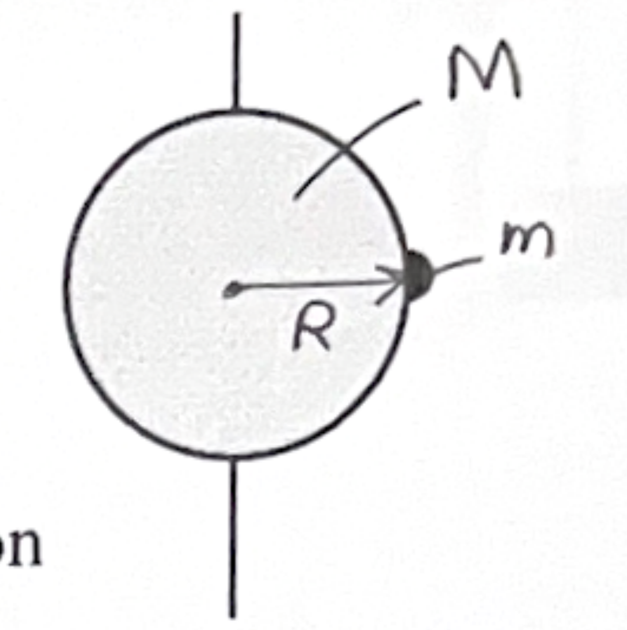
c) find the speed of the lowest point on the cylinder

$V_t = r\omega$

$V_t = (2R)(\omega) = (2)(.3 m)(6.6 \text{ rad/s}) = \boxed{3.96 \text{ m/s}}$

The lowest point is a distance 2R from the axis of rotation.

3. A uniform solid sphere of mass 5.5 kg and radius 30 cm is constrained to rotate about a vertical axis though its center. A small dense ball of mass 2.5 kg is attached to the edge of the sphere, at the equator. A constant force of 25.0 N is applied tangentially to the sphere at its equator. The sphere was at rest before the force began to be applied.



a) Circle the option in each row that best describes this process:

Constant net torque | Changing net torque

Constant angular acceleration | Changing angular acceleration

b) Find the rotational inertia of the system consisting of the sphere and the ball.

$I = I_{sphere} + I_{ball}$

$= \frac{2}{5}MR^2 + mR^2$

$= \frac{2}{5}(5.5kg)(.3m)^2 + (2.5kg)(.3m)^2$

$= 0.198 + .225$

$= \boxed{.423\ kg \cdot m^2}$

c) Find the angular acceleration of this system

$\boxed{\Sigma\tau = I\alpha}$ or $\boxed{\text{Kinematics}}$? I don't have enough kinematics info, so I'm using $\Sigma\tau = I\alpha$.

one torque
$\tau = rF\sin\phi$
$= (.3m)(25.0N)\sin 90°$
$= 7.5Nm$

$7.5\ Nm = (.423\ kg \cdot m^2)(\alpha)$

$\boxed{17.7\ rad/s^2 = \alpha}$

d) Find the angular displacement of the system during the first 10.0 seconds.

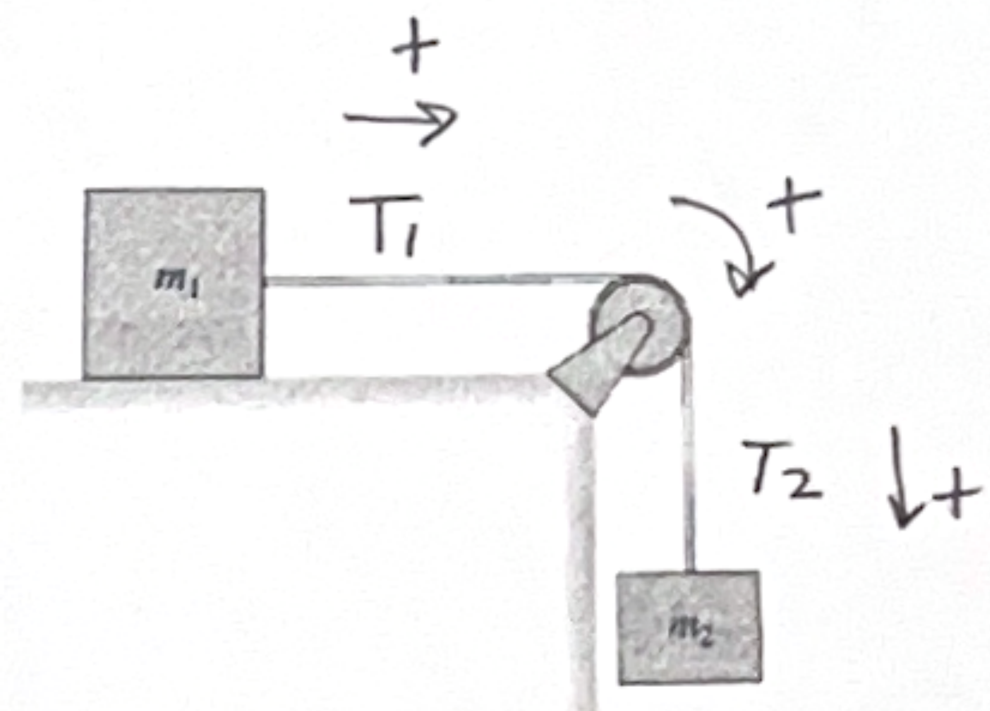
$\Delta\theta = ?$
$\omega_i = 0$
$\omega_f =$
$\alpha = 17.7\ rad/s^2$
$\Delta t = 10.0s$

$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2$

$\Delta\theta = 0(\Delta t) + \frac{1}{2}(17.7\ rad/s^2)(10.0s)^2$

$\boxed{\Delta\theta = 885\ rad}$

4. Blocks of mass m_1 and m_2 are connected by a massless string that passes over the pulley in the figure. Assume $m_1 > m_2$. Mass m_1 slides on a surface with negligible friction. The pulley has mass M and radius R, and its axle is frictionless. The tension in the horizontal string is T_1 and the tension in the vertical string is T_2. The linear acceleration of both blocks is a, and the angular acceleration of the pulley is α.

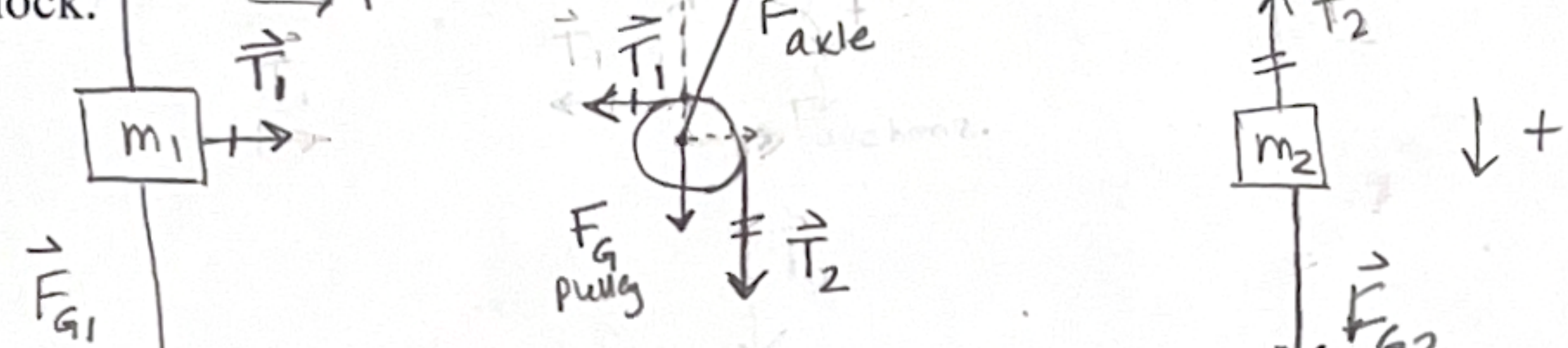


a) Circle the option in each row that best describes the pulley during this process:

Constant net torque — Changing net torque

Constant angular acceleration — Changing angular acceleration

b) Draw a qualitatively-correct FBD for each block and the pulley. Include all the forces. Label which direction is + for the rotation of the disk, and which direction is + for the acceleration of each block.



c) How must the tension T_2 compare to the force of gravity on block m_2? Why?

$T_2 < F_{G2}$ because m_2 has an acceleration vector that is downward.

d) How must the tension T_2 compare to the tension T_1? Why?

$T_2 > T_1$ because the pulley is accelerating, so it must be experiencing a net torque. Since its accel is CW, and R is the same for both forces, $T_2 > T_1$.

e) Apply Newton's second law in rotation form ($\Sigma\tau = I\alpha$) to the pulley. Your equation should be in terms of given variables and fundamental constants.

$\Sigma\tau = I\alpha$

$RT_2 \sin 90 - RT_1 \sin 90 = (\frac{1}{2}MR^2)\alpha$

$\boxed{T_2 - T_1 = (\frac{1}{2}MR)\alpha}$

f) Apply Newton's second law (Fnet = ma) to write an equation for the horiz. motion of block on the table. Your equation should be in terms of given variables and fundamental constants.

$\Sigma F = ma$

$\boxed{T_1 = m_1 a}$

g) Apply Newton's second law (Fnet = ma) to the falling block. Your equation should be in terms of given variables and fundamental constants.

$\Sigma F = ma$

$F_{G2} - T_2 = m_2 a$

$\boxed{m_2 g - T_2 = m_2 a}$

h) Write an equation that relates the angular acceleration of the pulley to the linear acceleration of the blocks. Your equation should be in terms of given variables and fundamental constants.

The tangential acceleration of the disk is the same as the linear acceleration of each block, so $a_t = a$.
And the tangential acceleration of the disk is related to its angular acceleration by $a_t = \alpha R$. Therefore, $\boxed{a = \alpha R}$