Unit 8 Practice: Answers and Solutions (P15 to P23)

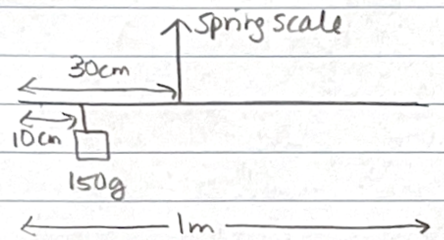
Your instructor greatly appreciates reports of any mistakes you find in these documents. ☺

P15) An object will be in static equilibrium if both of these conditions are met:
The sum of the forces on the object is zero ($\Sigma\vec{F} = 0$)
The sum of the torques on the object is zero ($\Sigma\vec{\tau} = 0$)

P16)

P16)

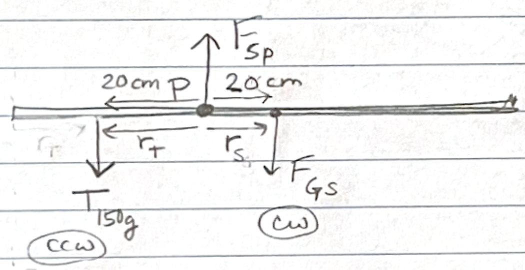
a) Sketch + translate



b) Simplify and diagram

system: meterstick

models: rigid body, static equilibrium



Sp = spring scale
S = stick
T = tension
m_s = mass of stick

c) Represent mathematically

$\Sigma\tau = 0$

$r_T T_{150g} \sin 90° - r_S F_{GS} \sin 90° = 0$

$(.2m)(.150kg)(9.8 N/kg) - (.2m)(m_s) = 0$

$\boxed{150g = m_s}$

You might have seen this result based on the symmetry.

$\Sigma F_y = 0$

$F_{Sp} - T_{150g} - F_{GS} = 0$

$F_{Sp} = 2(150g)(9.8 N/kg)$

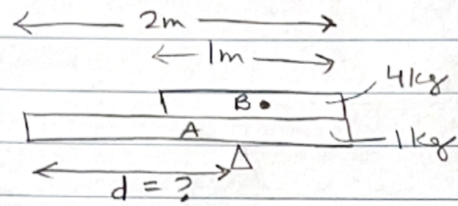
$= \boxed{2.9N}$

(e) 150 g is reasonable for a wooden meterstick, and 2.9N is similar to forces we measured in class for similar scenarios

P17) p.331 #29

p. 331 #29

a) Sketch + translate



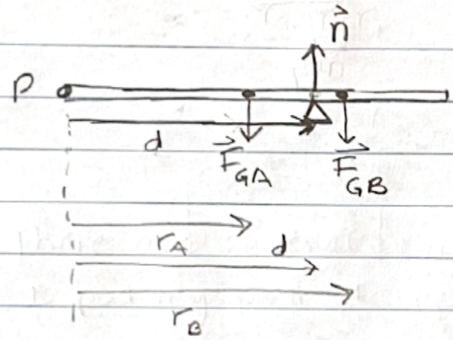
Find d

Assume C.M. of the block B is at its center, so 0.5m from end of plank A.

b) simplify + diagram

object: bottom rectangular beam

models: rigid body, static equilibrium



$F_{GA} = m_A g = (1kg)(9.8 N/kg) = \boxed{9.8N}$

$F_{GB} = m_B g = (4kg)(9.8 N/kg) = \boxed{39.2N}$

$\Sigma \vec{\tau} = 0$

$\vec{\tau}_A + \vec{\tau}_B + \vec{\tau}_n = 0$

$-(r_A)(F_{GA})\sin 90° - (r_B)(F_{GB})\sin 90° + (d)(n)\sin 90° = 0$

$-(1m)(9.8N)(1) - (1.5m)(39.2N)(1) + (d)(n)(1) = 0$

$-9.8 - 58.8 + nd = 0$

$-68.6 + nd = 0$ ⟶ I need to find n!

$\Sigma F_y = 0$

$n - F_{GA} - F_{GB} = 0$

$n = 9.8N + 39.2N$

$n = \boxed{49N}$

$-68.6 + (49N)d = 0$

$-68.6 = -49d$

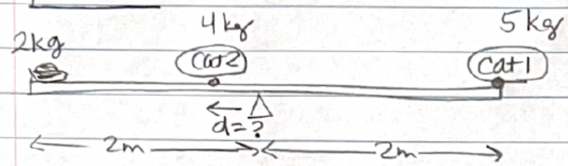
$\boxed{1.4m = d}$

(e) 1.4 m is reasonable for the pivot (balance) point because we would expect it to be right of the center of the plank (1m), but left of the C.M. of the block B (1.5m).

P18) p.331 #32

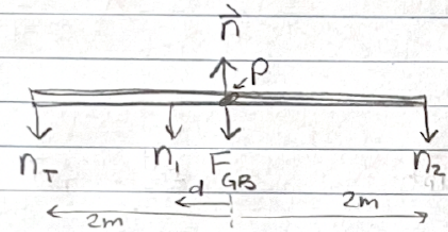
p.331 #32

(a)



(b) System: seesaw beam

models: rigid body, static equilibrium



1 = cat1

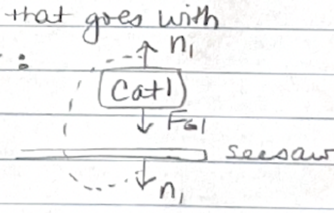
2 = cat2

B = seesaw beam

T = tuna

Note: Because the tuna bowl and cats are not part of my system, their mass is not part of my system, so earth does not exert a force on their mass in my system. Therefore I don't show F_G forces on my FBD for those objects (although I have done that in previous problems, but shouldn't have!)

The force on the seesaw from objects on the seesaw is a normal force. It is the "reaction" (N3L) force that goes with the upward force of the beam on the object:



And you can see that $n_1 = F_{G1}$ since the cat is at rest.

So... $n_T = F_{GT} = (2kg)(9.8 N/kg) = 19.6N$

$n_1 = F_{G1} = (4kg)(9.8 N/kg) = 39.2N$

$n_2 = F_{G2} = (5kg)(9.8 N/kg) = 49 N$

(c) represent mathematically

(d) solve

$$\Sigma \vec{\tau} = 0$$

$$r_T n_T \sin 90 + r_1 n_1 \sin 90 - r_2 n_2 \sin 90 = 0$$

$$(2m)(19.6N)(1) + (d)(39.2N)(1) - (2m)(49N)(1) = 0$$

$$39.2Nm + (39.2N)d = 98\,Nm$$

$$(39.2N)d = 58.8\,Nm$$

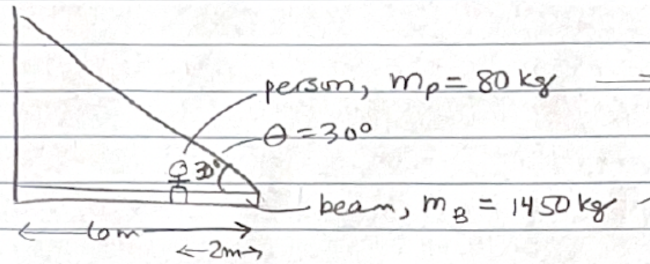
$$d = \boxed{1.5m}$$

(e) 1.5m makes sense because it should be less than 2m, which is at the center of the beam, and we would expect this smaller cat to not be at the tuna bowl, because that would make 6kg at the left end which would not balance the 5 kg cat at the other end.

P19) p.333 #59

p. 333 #59

a)

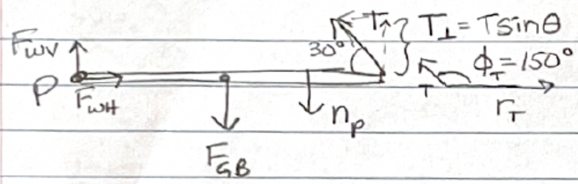


$F_{Gp} = m_p g = (80\,kg)(9.8\,N/kg) = \boxed{784N}$

$F_{GB} = m_B g = (1450\,kg)(9.8\,N/kg) = \boxed{14210\,N}$

b) System: beam

models: rigid body, static equilibrium



F_{WV} = vertical force from wall

F_{WH} = horiz. force from wall

T = tension

n_p = normal force from person

c) Represent mathematically

$n_p = F_{Gp}$ because $a_y = 0$

$$\Sigma \vec{\tau} = 0$$

$$-r_B F_{GB} \sin 90° - r_p n_p \sin 90° + r_T (T)_\perp = 0$$

$$-(3m)(14210N)(1) - (4m)(784N)(1) + (6m)(T\sin 30°) = 0$$

$$45766 = 3T$$

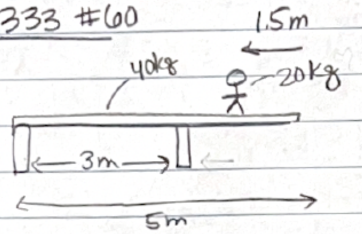
$$\boxed{15300N = T}$$

(e) This answer makes sense because the beam is quite heavy, so a large tension force is reasonable.

P20) p.333 #60

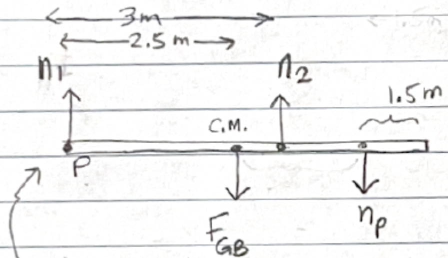
p.333 #60

(a)



(b) system: beam

models: rigid body, static equilibrium



Suppose I don't actually know yet if n_2 is up or down. So I guess. If I'm right, n_2 will come out + in my calculations. If I'm wrong, it will come out −.

$F_{GB} = (40\text{ kg})(9.8\text{ N/kg}) = 392\text{N}$

n_p = normal force from person

$n_p = F_{Gp} = (20\text{ kg})(9.8\text{ N/kg}) = 196\text{ N}$

because the upward n_p on the boy is equal to the downward n_p on the beam, and $F_{Gp} = n_p$ because the boy is at rest, so $\Sigma F_y = 0$ on him.

(c)(d)

with the pivot at the left support: $\Sigma \vec{\tau} = 0$

$$-(2.5\text{ m})(F_{GB})\sin 90° + (3\text{ m})(n_2) - (3.5\text{m})n_p = 0$$

$$-(2.5)(392\text{N})(1) + 3n_2 - 3.5(196\text{N}) = 0$$

$$3n_2 = 980 + 686$$

$$\boxed{n_2 = 555\text{ N, up}}$$

Now I can find n_2:

$\Sigma F_y = 0$

$n_1 + n_2 - F_{GB} - n_p = 0$

$n_1 + 555\text{N} - 392\text{N} - 196\text{N} = 0$

$n_1 = 33\text{N}$, so I assumed correctly that n_1 is up. $\boxed{n_1 = 33\text{ N, up}}$

(e) $n_2 \gg n_1$, which makes sense because n_2 is very close to F_{GB} and n_p, as well as between them, so n_2 is supporting most of their weight

P21) p.313 Example 12.15. Here are the forces on the ladder:

*Answers are in terms of the mass of the ladder, m_L, since no value was given for that.

p.313, Ex 12.15 : All forces on the ladder

L = Ladder

m_L = mass of ladder

$\Sigma F_x = 0$

$n_2 - f_s = 0$

$n_2 = f_s$

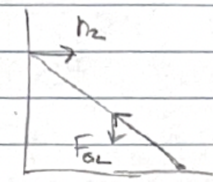
$\Sigma F_y = 0$

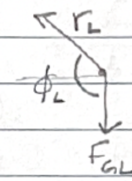
$n_1 - F_{G} = 0$

$n_1 = F_g$

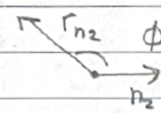
$\Sigma \tau = 0$

$r_L F_{GL} \sin\phi_L - r_{n_2} n_2 \sin\phi_{n_2} = 0$





$\phi_L = 90 + 60 = 150°$



$\phi_{n_2} = 180 - 60 = 120°$

$(1.5\,m)(m_L)(9.8)\sin 150 - (3\,m)\,n_2 \sin 120 = 0$

$\frac{7.35\,m}{2.598} = n_2$

$2.8\,m_L = n_2$

$2.8\,m_L = f_s$

$n_1 = m_L(9.8)$

$n_1 = 9.8\,m_L$

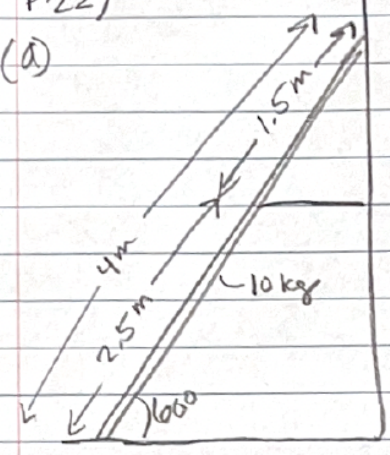
$F_{GL} = m_L(9.8)$

$F_{GL} = 9.8\,m_L$

P22)

P22)

(a)

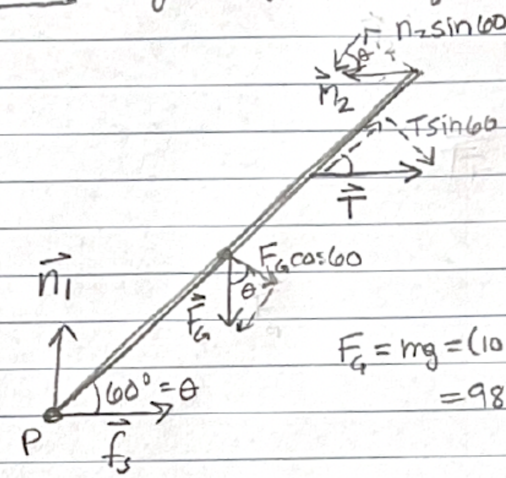


$\mu_s = .1$

max static friction must be occurring since the rope is providing the extra force that f_s couldn't provide b/c it couldn't get any larger.

(b) system: board

models: rigid body, static equilibrium



$F_G = mg = (10\,kg)(9.8\,\frac{N}{kg})$
$= 98N$

(c) represent mathematically

$$\Sigma \tau = 0$$

$$-r_{FG}(F_G)_\perp - r_T(T)_\perp + r_{n_2}(n_2)_\perp = 0$$

$$-(2m)(98N)\cos 60 - (2.5m)T \sin 60 + (4m)n_2 \sin 60 = 0$$

Eqn ① ⇒ $-98 - 2.17T + 3.46\,n_2 = 0$

I have 2 unknowns, so I need another equation!

$$\Sigma F_x = 0$$

$$f_{s\,max} + T - n_2 = 0$$

$$9.8N + T - n_2 = 0$$

$$9.8 + T = n_2$$

$$\Sigma F_y = 0$$

$$n_1 - F_G = 0$$

$$n_1 = 98N$$

$$f_{s\,max} = \mu_s n_1$$
$$= (.1)(98N)$$
$$= 9.8N$$

Now I can put this in for n_2 in Eqn ①

Eqn ① ⇒ $-98 - 2.17T + 3.46\,n_2 = 0$

$-98 - 2.17T + 3.46(9.8 + T) = 0$

$-98 - 2.17T + 33.9 + 3.46T = 0$

$-64.1 + 1.29T = 0$

$-64.1 = -1.29T$

$\boxed{49.7\text{N} = T}$

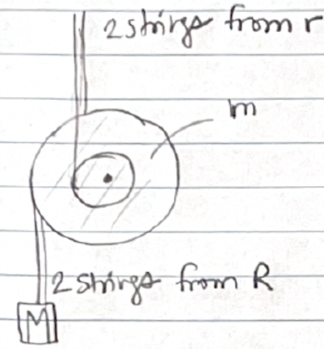
The problem says the answer is 50.3N. I think there are rounding errors.
49.7N ≈ 50.3 N

e) 49.7N seems reasonable for a horizontal rope helping 9.8 N of friction keep a 10kg tilted board from sliding.

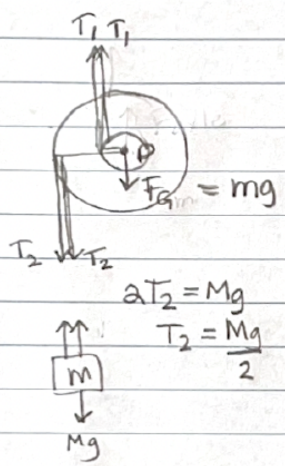
P23)

P23)

a)



b)



$2T_2 = Mg$

$T_2 = \frac{Mg}{2}$

System: the spool

models: rigid body, static equilibrium

pivot is at the center of mass of the spool

(c)(d)

$\Sigma F_y = 0$

$2T_1 - mg - 2T_2 = 0$

$2T_1 - mg - 2\left(\frac{Mg}{2}\right) = 0$

$2T_1 - mg - Mg = 0$

$T_1 = \frac{(m+M)g}{2}$

$\Sigma \vec{\tau} = 0$

$2(R)T_2 - 2rT_1 = 0$

$2R\left(\frac{Mg}{2}\right) - 2r\left(\frac{m+M}{2}\right)g = 0$

$RMg - r(m+M)g = 0$

Find M: $RMg - rmg - rMg = 0$

$M(Rg - rg) = rmg$

$M = \frac{rmg}{(Rg - rg)}$

$M = \frac{rmg}{(R-r)g}$

$M = \frac{mr}{R-r}$

(e) This makes sense because if I think of the edge at r as the pivot, and I rewrite my result as $M(R-r) = mr$, each side is like a torque, where distance to Mg is R−r, distance to mg is r.