

Angular Position and Velocity

Coordinate system: Positive angles are CCW, so positive angular velocity is CCW

1. What is the definition of a radian? (see p.93)

$$\theta \equiv \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$



2. The tip of the second hand moves in a circular path of radius 4.0 cm. (a) When the tip has moved a distance of 4.0 cm along its circular path, what angle has it moved through in radians? (b) What about when the tip has moved a distance of 8.0 cm along its circular path?

a) $\Delta\theta = \frac{\Delta s}{r} = \frac{4.0\text{cm}}{4.0\text{cm}} = 1\text{ rad!}$ b) For 8cm, $\Delta\theta = \frac{\Delta s}{r} = \frac{8\text{cm}}{4\text{cm}} = 2\text{ rad}$

3. The magnitude of the radial acceleration is $a_r = v^2/r$ in terms of an object's tangential speed and the radius of its circular path. Show that in terms of an object's angular speed, the magnitude of the radial acceleration is $a_r = \omega^2 r$.

ω and v are related by $v = \omega r$

$$a_r = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$$

4. The blade of a certain blender rotates at 3000 rpm in the clockwise direction on low speed. The diameter of the blade is 4.0 cm.

a) What is the angular velocity of the blade in rad/s? What is it in degrees/s?

$$\omega = 3000 \frac{\text{rev}}{\text{min}}$$

$$|\omega| = \frac{2\pi}{T} = \frac{2\pi}{0.02\text{s}} = 100\pi \text{ rad/s} = 314 \text{ rad/s}$$

So it makes 3000 rev/60s = 50 rev in 1 second,

$$\omega = \frac{360^\circ}{T} = \frac{-360^\circ}{0.02\text{s}} = -18000^\circ/\text{s}$$

b) What is the period of the blade? so it takes 1/50 s

for one Rev, = 0.02s

$$T = 0.02\text{s}$$

↖ work is up here

* negative because clockwise!

c) What is the blade's angular position at $t = 3.0$ minutes if its initial position was 0 radians?

$$\omega = \frac{\Delta\theta}{\Delta t}$$

↳ x60s = 180s

$$\Delta\theta = \omega \Delta t = (-314 \text{ rad/s})(180\text{s}) = -56520 \text{ rad}$$

d) How many complete revolutions did the blade make during the 3.0 minutes?

$2\pi \text{ rad}$ for every complete rev, so...

$$\# \text{ rev} = \frac{56520 \text{ rad}}{2\pi} = 8995.44, \text{ so it made } 8995 \text{ complete revolutions and } .44 \text{ of another one}$$

e) Where is the blade located between 0° and 360° at the end of the 3.0 minutes?

It made 0.44 of the last revolution

$$\text{So, } 0.44 \text{ of } 360^\circ = .44 \times 360^\circ = 158^\circ$$

But it is going CW, so it is at -158° or $+202^\circ$.

f) What is the tangential speed of a speck of soup on the tip of the blade?

$$v = \omega r$$

$$= (-314 \text{ rad/s})(0.02\text{m})$$

$$= -6.28 \text{ m/s}$$

Speed is 6.28 m/s

