Circular Motion Practice

1. A bucket of water of mass M is tied to the end of a string and whirled in a vertical circle of radius R. Suppose the whirler keeps the bucket moving at a constant speed of v_o. Find the tension in the string when the ball is at its lowest point in terms of given variables and fundamental constants, showing the steps below.

    a) **Sketch and translate**
    b) **Simplify and diagram**
    c) **Represent mathematically**
    d) **Solve**
    e) If the speed of the bucket was doubled, and the radius of the path was quadrupled, how would this change the tension in the string at the lowest point?
    f) How would the force of tension in the string compare at the top and bottom of the circle? Why?

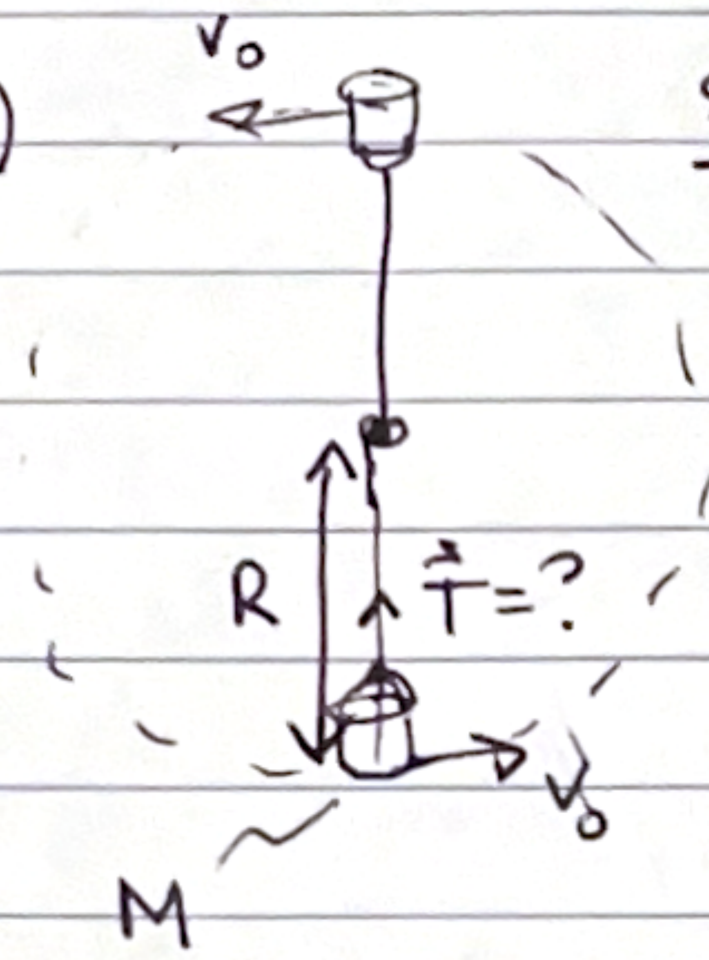
2. A car is rounding a curve of radius R on a flat road. The coefficient of static friction is μ_s, the mass of the car is M, and it is traveling with speed v_o. Find the static friction force needed for the car to make the turn in terms of given variables and fundamental constants, showing the steps below.
    a) **Sketch and translate**
    b) **Simplify and diagram**
    c) **Represent mathematically**
    d) **Solve**
    e) If the mass of the car is doubled, how would this change the magnitude of the static friction force needed?

3. A car is rounding a curve of radius R on a flat road. The coefficient of static friction is μ_s, the mass of the car is M, and it is going at the maximum possible speed v_{max} to make the turn. Find the car's maximum speed (symbolically). Show the steps below.
    a) **Sketch and translate**
    b) **Simplify and diagram**
    c) **Represent mathematically**
    d) **Solve**
    e) If the radius of the curve was quadrupled, how would this change the maximum speed?

4. A satellite (mass m) orbits the earth (mass M) at constant orbital speed v_o in a circular path which has a radius R. Find the orbital speed of the satellite (v_o) in terms of given variables and fundamental constants. Show the steps below.
    a) **Sketch and translate**
    b) **Simplify and diagram**
    c) **Represent mathematically**
    d) **Solve**
    e) If the radius of the orbit was quadrupled, how would this change the satellite's orbital speed?

Circular Motion Practice

1. a)



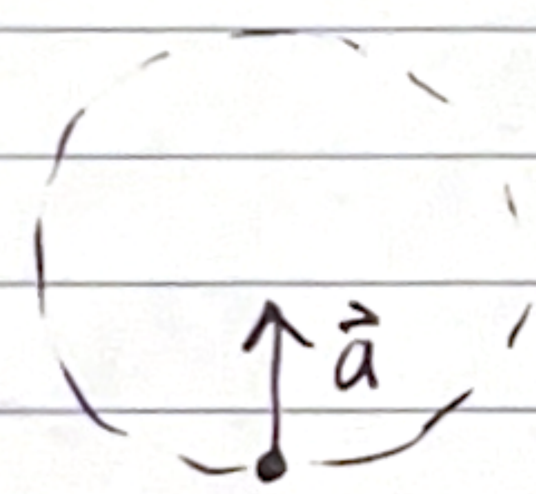
Sketch + translate

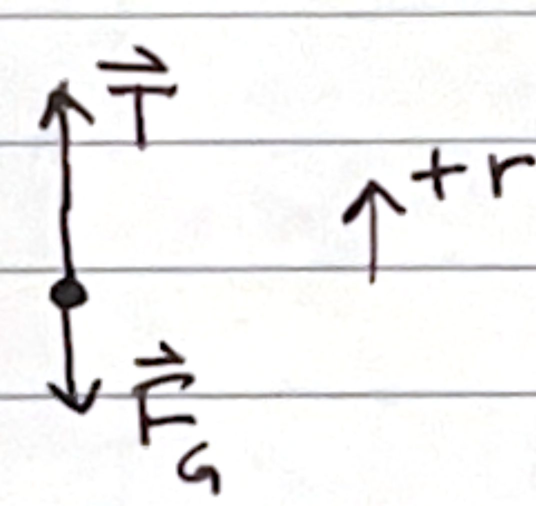
System: Bucket of water

position: Bottom of circle

b) **Simplify and Diagram**

- model bucket as a particle
- model motion as uniform circular motion





The tension force has greater magnitude than F_G.

c) **Represent mathematically**

$$\Sigma F_r = ma_r$$

$$T - F_G = m\left(\frac{v^2}{r}\right)$$

$$T - Mg = M\left(\frac{v_0^2}{R}\right)$$

d) **Solve**

I am finding the tension

$$\boxed{T = Mg + \frac{Mv_0^2}{R}}$$

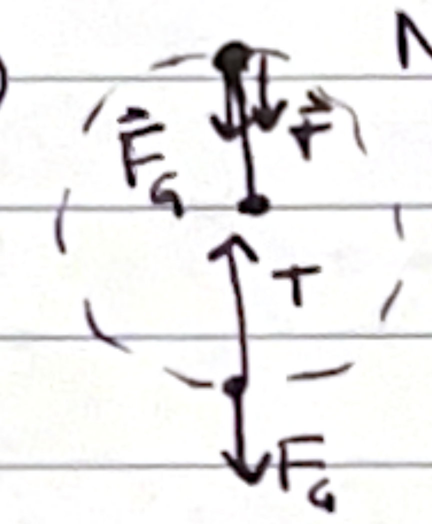
e) If $v_2 = 2v_0$, and $r_2 = 4R$, then the acceleration, $a_r = \frac{v^2}{r}$, is

$$a_r = \frac{(2v_0)^2}{4R} = \frac{4v_0^2}{4R} = \frac{v_0^2}{R}$$

So the acceleration stays the same, so the tension is still $T = Mg + Mv_0^2/R$, so the tension does not change.

f) $\Sigma F_r = ma_r$

at top;



Newton's second law is $F_G + T = ma_r$

at bottom, Newton's second law is $T - F_G = ma_r$

Both cases must have the same net force because the mass and acceleration are the same. Looking at the FBD, we can conclude that the tension is greater at the bottom because the net force is $T - F_G$, whereas at the top the tension and the force of gravity add together to make the net force. Does that make sense?

2) a) Sketch + translate

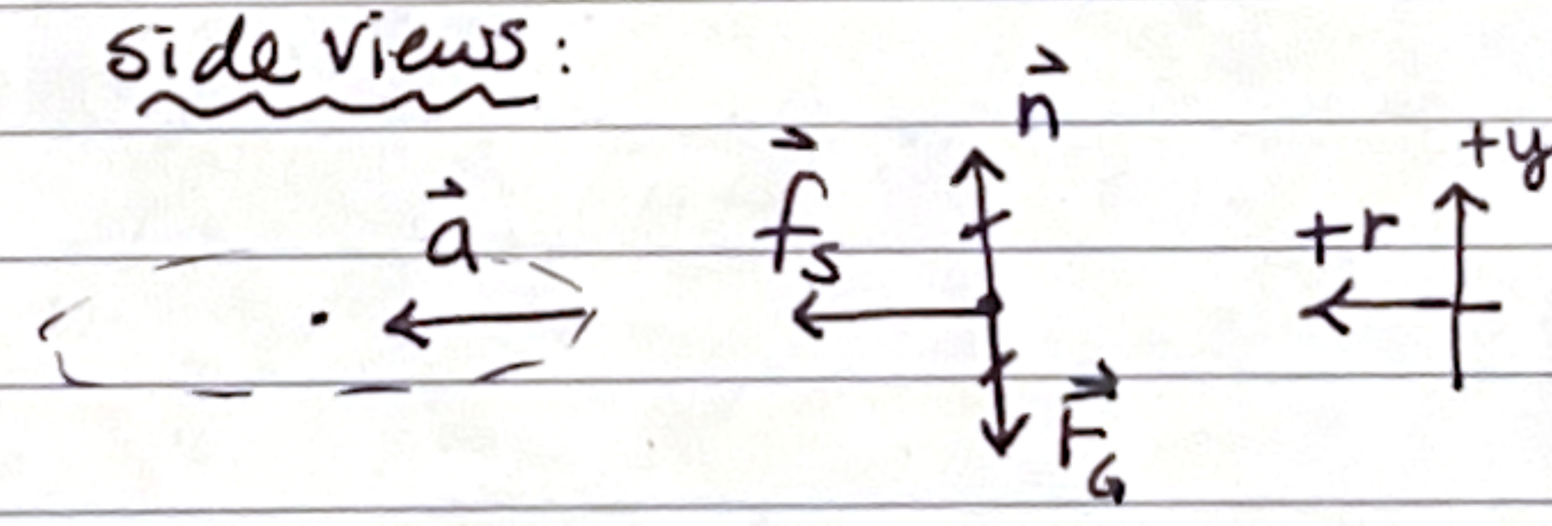
top view

System: car

b) Simplify and Diagram

- model car as particle
- model motion as uniform circular motion

side views:



Represent mathematically

c) $\Sigma F_r = ma_r$

$f_s = \dfrac{mv^2}{r}$

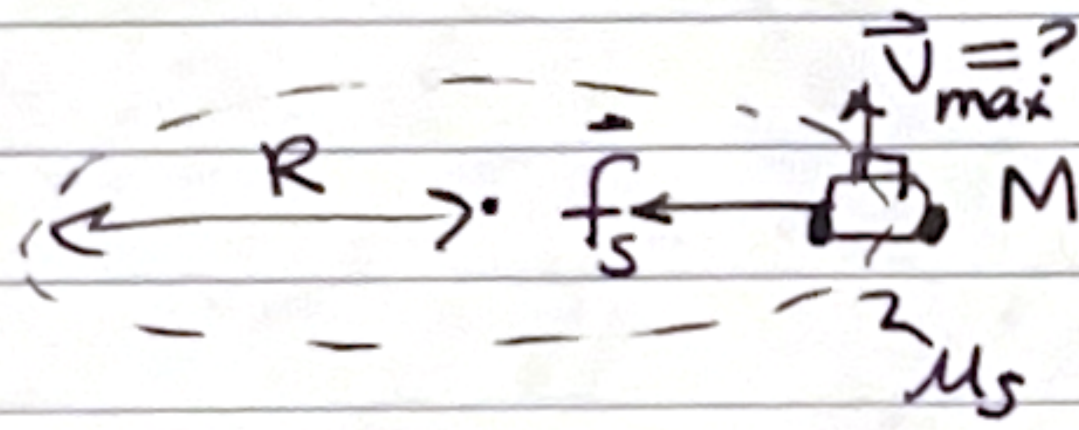
$\boxed{f_s = \dfrac{mv_0^2}{R}}$

We cannot use $f_s = \mu_s n$ to find the static friction force in this situation because we have no indication that the static friction force is a maximum, and that equation tells you the maximum.

d) Solve (Done!)

e) The friction force needed is given by $f_s = \dfrac{mv_0^2}{R}$. In this equation, if we double the mass, the friction force also doubles.

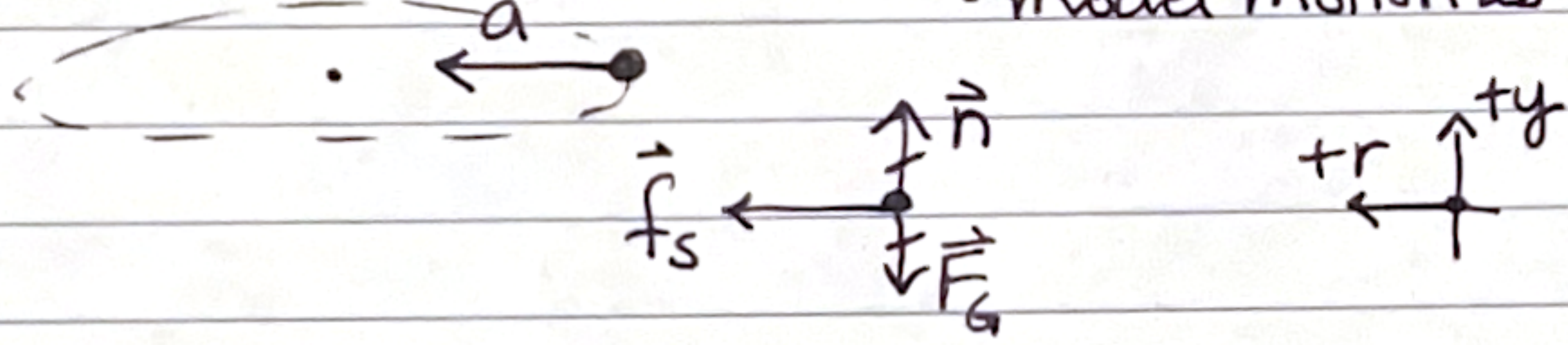
③ a) Sketch and Translate



System: car
position: as shown

b) Simplify and Diagram

- model car as particle
- model motion as uniform circular motion



c) Represent mathematically

$\Sigma F_r = ma_r$
$f_s = m\left(\frac{v^2}{r}\right)$

Since the car is going at the max possible speed, we know the static friction force is at its maximum value of $f_s = \mu_s n$!

$\therefore \mu_s n = M\left(\frac{V_{max}^2}{R}\right)$

The normal force, n, is not a given variable, so I can't have it in my answer. H could I find it? From applying $\Sigma F = ma$ in the y-direction:

$\Sigma F_y = ma_y$
$n - F_G = m(0)$
$n = F_G$
$n = Mg$

$\mu_s (Mg) = M\left(\frac{V_{max}^2}{R}\right)$

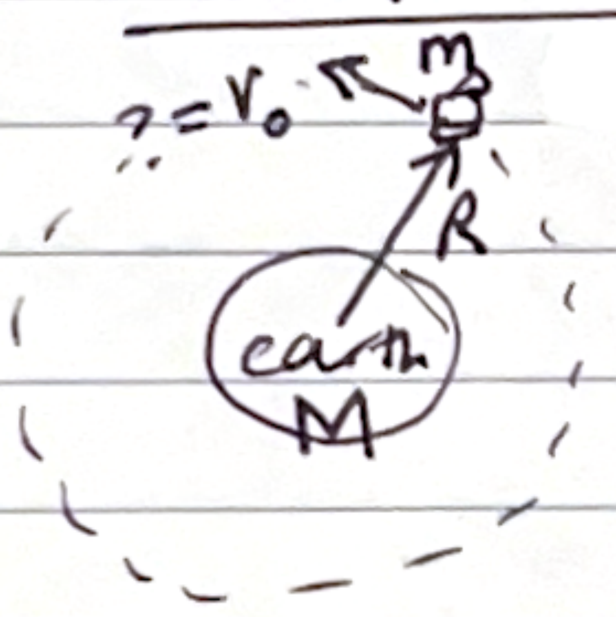
$\mu_s g R = V_{max}^2$

$\boxed{\sqrt{\mu_s g R} = V_{max}}$

d) Solve (Already done!)

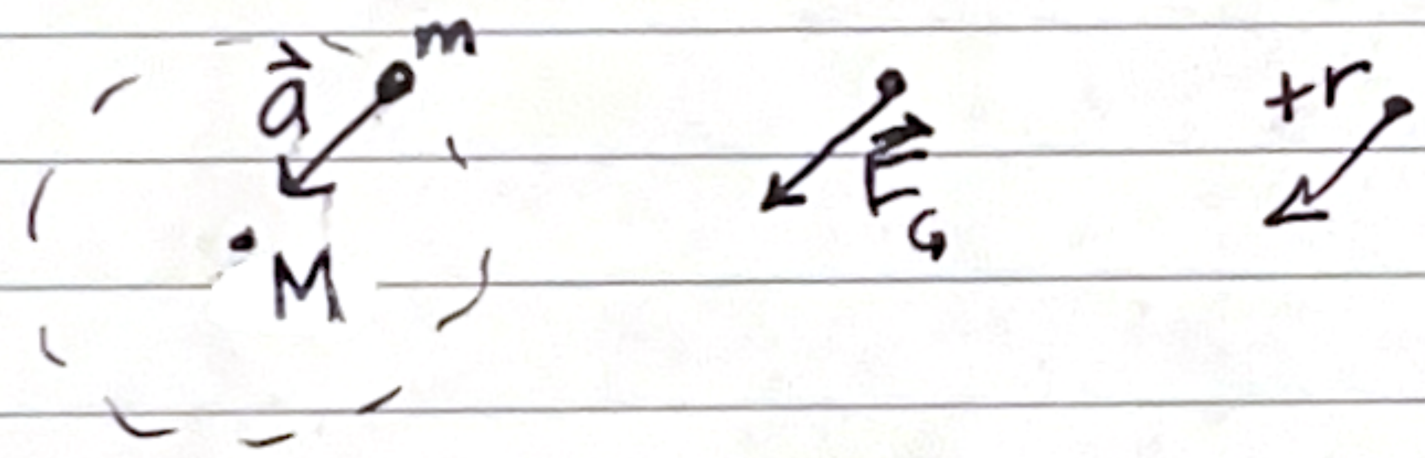
e) The maximum velocity is $\sqrt{\mu_s g R}$. So if the new radius is $r_2 = 4R$, $V_{max_2} = \sqrt{\mu_s g (4R)} = 2\sqrt{\mu_s g R} = 2V_{max}$. So the maximum speed doubles.

④ a) Sketch + translate



System: satellite
position: the point drawn

b) Simplify + diagram
- model satellite as particle
- model process as circular motion at constant speed



c) Represent mathematically

$$\Sigma F_r = ma_r$$

Because the satellite is not near the earth's surface, we can't use $g = 9.8 \, N/kg$. We need to use $F_G = G\frac{m_1 m_2}{r^2}$ for the force of gravity.

$$\therefore F_G = m\frac{v^2}{r}$$

$$\frac{GMm}{R^2} = m\frac{v_0^2}{R}$$

This is the mass of the object that is moving in the circular path -- the one that has the acceleration.

$$\frac{GM}{R} = v_0^2$$

$$\sqrt{\frac{GM}{R}} = v_0$$

e) If the radius is quadrupled, $r_2 = 4R$, and $v_{0_2} = \sqrt{\frac{GM}{4R}} = \frac{1}{2}\sqrt{\frac{GM}{R}}$, so the orbital speed becomes $\frac{1}{2}$ of what it was before.