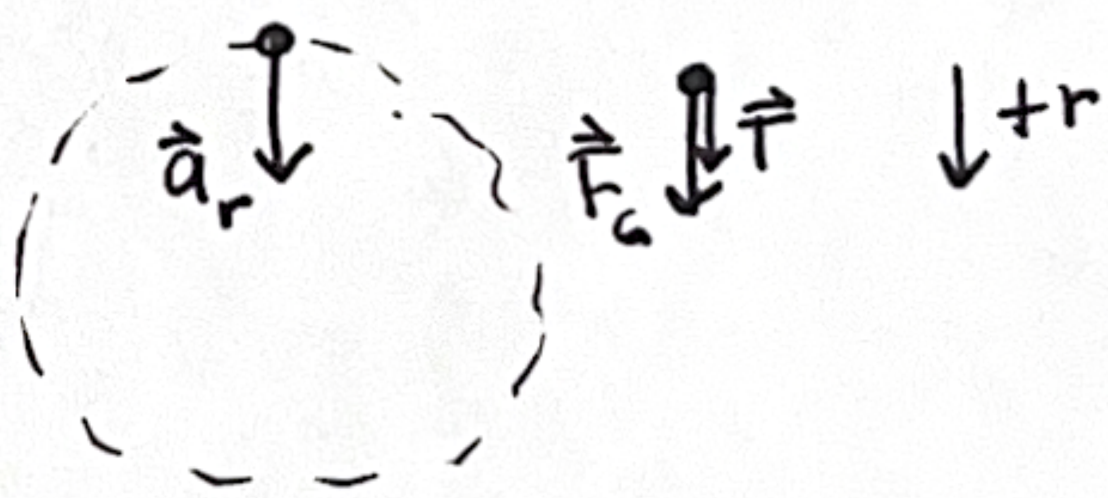
Circular Motion Problem-Solving

This general approach is useful when analyzing situations of circular motion: Begin with a sketch of the situation and choose a specific position to analyze. Then determine the direction of the object's acceleration as it passes the position you chose and draw the forces on the object at that point. Finally, define the direction of your +r-axis and apply Newton's second law for the radial direction, which is $\Sigma F_r = ma_r$. Remember that the radial acceleration is given by $a_r = \frac{v^2}{r}$. (In some situations, you will also need to apply Newton's second law in the y-direction, $\Sigma F_y = ma_y$.)

Example : A ball of mass m is twirled in a vertical circle by a string at a constant speed of v_o. The string has length L. What is the tension T in the string when the ball is at the top of the circle?

Sketch and translate • Sketch the situation described. Label all relevant information. • Choose the system and a specific position to analyze. • Identify the unknown that you need to find and label it with a question mark.	
Simplify and diagram • Decide if the system can be modeled as a point-like object. • Determine if the constant speed circular motion approach is appropriate. • Indicate with an arrow the direction of the object's acceleration as it passes the chosen position. • Draw an FBD for the system at the instant it passes that position. • Beside the FBD, draw your +r-axis	— yes — yes 
Represent mathematically • Convert the force diagram into the radial r-component form of Newton's second law. • For objects moving in a horizontal circle you may also need to apply a vertical y-component form of Newton's second law	$\Sigma F_r = ma_r$ $F_G + T = m\left(\frac{v^2}{r}\right)$ $mg + T = m\left(\frac{V_o^2}{L}\right)$
Solve • Solve the equations formulated in the previous step.	$\boxed{T = m\left(\frac{V_o^2}{L}\right) - mg}$

If the length of the string was increased, and everything else stayed the same, how would the tension in the string change?

If L increases, the term $\frac{mV_o^2}{L}$ will decrease. Since $T = \frac{mV_o^2}{L} - mg$, if that first term $\frac{mV_o^2}{L}$ gets smaller, $\frac{mV_o^2}{L} - mg$ also gets smaller. Therefore the tension will decrease.

Static Friction in Circular Motion

Sometimes the force of static friction is involved in circular motion. When a car rounds a curve on a flat road, what force could be providing the net force toward the center of the circle? Static friction! To help grasp this, imagine what would happen if there was no friction between the tires and the road. Can you see that the car would just continue going in a straight line, because without any *sideways force* from the tires gripping the road, there would be no force than could change the *direction* of the car's velocity?

When working with the force of static friction, it is very important to remember that *the force of static friction can take on any value needed up to a maximum value*. For example, imagine a car that is going around a curve on a flat road with increasing speed. The acceleration of the car toward the center of the circle is $a_r = v^2/r$, so as the speed of the car increases, the radial acceleration increases. As the radial acceleration increases, the radial net force must also increase to keep the car on the circular path. The force of static friction opposes an object's the tendency to slide, so as the car's speed increases and there is a tendency to slide, the force of static friction will increase to oppose it. As the speed of the car keeps increasing, the force of static friction will keep increasing … until it reaches its maximum value! The maximum speed at which the car can make the turn occurs when the static friction force is at its maximum value. If the car tries to make the turn at a speed that is greater than this maximum speed, there will not be enough static friction force toward the center, and the car will not be able to make the turn.

Because the equation $f_{smax} = \mu_s n$ predicts the maximum value possible for the static friction force, it does not always tell you the actual magnitude of the static friction force that is occurring in a particular situation. Sometimes the static friction force has a magnitude that is less than its maximum value. The static friction force will occur at its maximum value in situations where a car is moving at the *maximum speed* possible to make a turn of a certain radius, or where a car is going around a turn with the *minimum radius* possible at a certain speed. In situations where the static friction force occurs with a value that is less than its maximum, you can find a value for the static friction force by applying Newton's 2nd Law, but not by using $f_{smax} = \mu_s n$.

Sometimes, the force of static friction will be in the vertical direction! For example, imagine a shirt whirling around during the spin cycle in a top-load washing machine. As the shirt goes around in a circle, a force is exerted on the shirt by the drum, and the direction of this force is toward the center of the circle. This is a normal force because it is caused by an interaction with a surface. There is also a downward force on the shirt due to its gravitational interaction with the earth; this is F_G. We note that the shirt is not accelerating in the vertical direction, so the net force must be zero vertically. What upward force on the shirt could be balancing the downward force of gravity? It is the force of static friction between the shirt and the drum!

Problem 1: A car of mass M traveling on a flat road goes around a curve of radius R with speed v_o. The coefficient of friction between the road and tires is μ_s. Find the force of static friction acting on the car in the radial direction in terms of given variables and fundamental constants.

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Sketch and translate

- Sketch the situation described. Label all relevant information.
- Choose the system and a specific position to analyze.
- Identify the unknown that you need to find and label it with a question mark.

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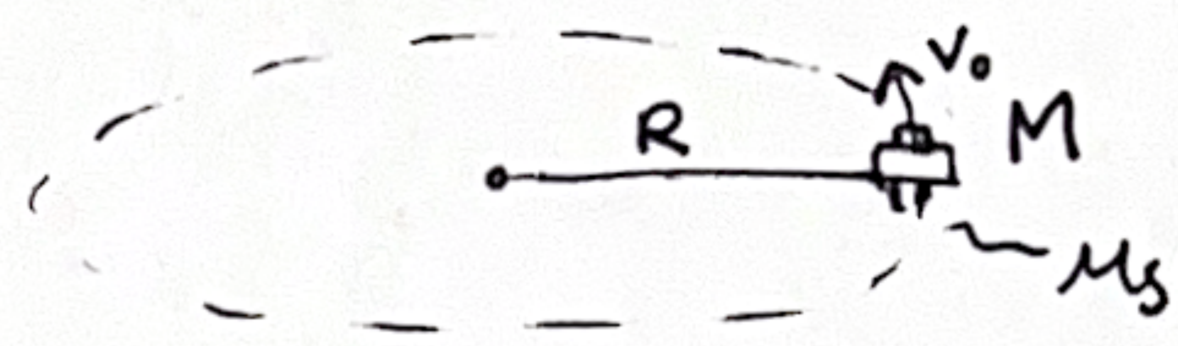

$f_s = ?$

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Simplify and diagram

- Decide if the system can be modeled as a point-like object. — yes
- Determine if the constant speed circular motion approach is appropriate. — yes
- Indicate with an arrow the direction of the object's acceleration as it passes the chosen position.
- Draw an FBD for the system at the instant it passes that position.
- Beside the FBD, draw your +r-axis

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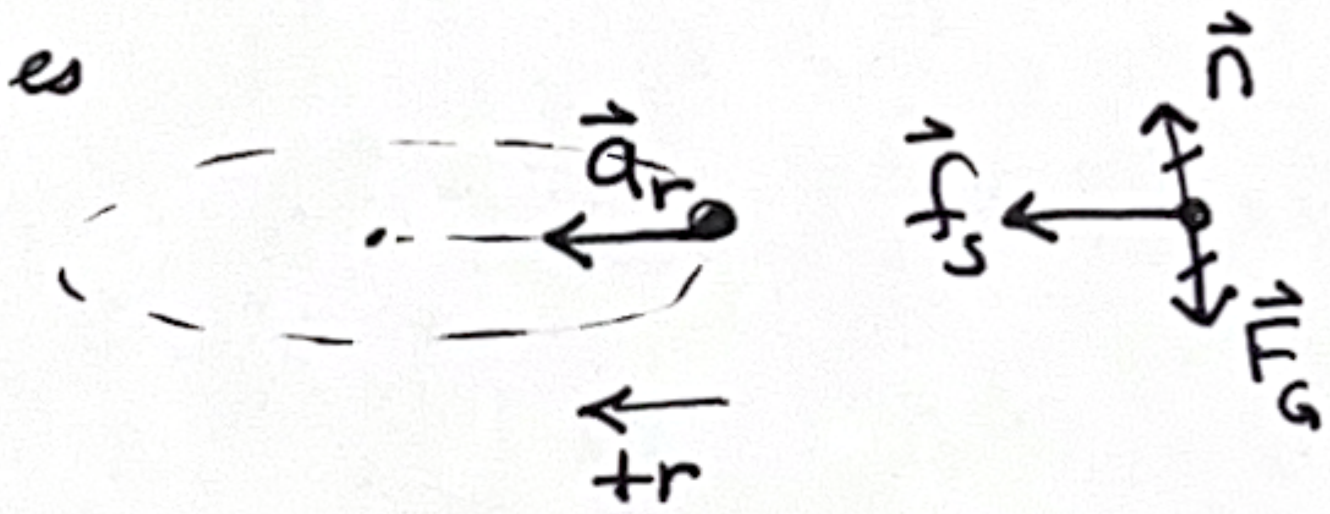

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Represent mathematically

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- For objects moving in a horizontal circle you may also need to apply a vertical y-component form of Newton's second law

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$$\Sigma F_r = ma_r$$
$$f_s = m\left(\frac{v^2}{r}\right)$$
$$f_s = M\left(\frac{V_o^2}{R}\right)$$

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Solve

- Solve the equations formulated in the previous step.

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$$\boxed{f_s = M\left(\frac{V_o^2}{R}\right)}$$

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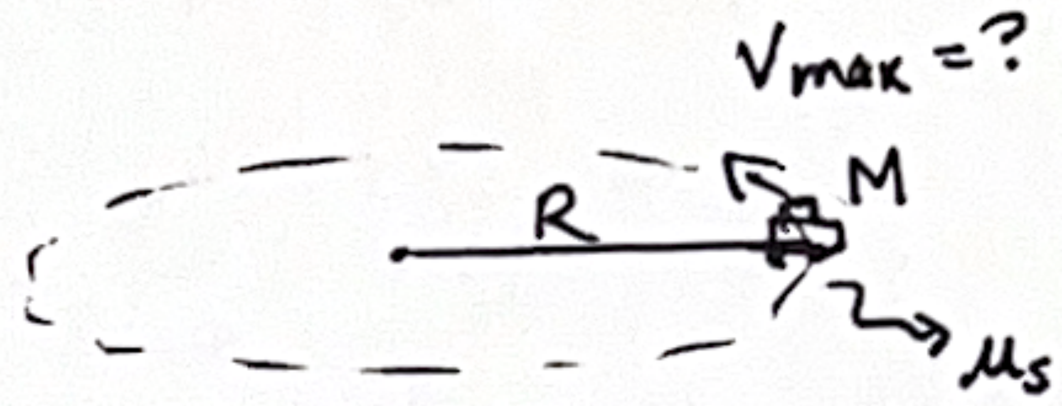
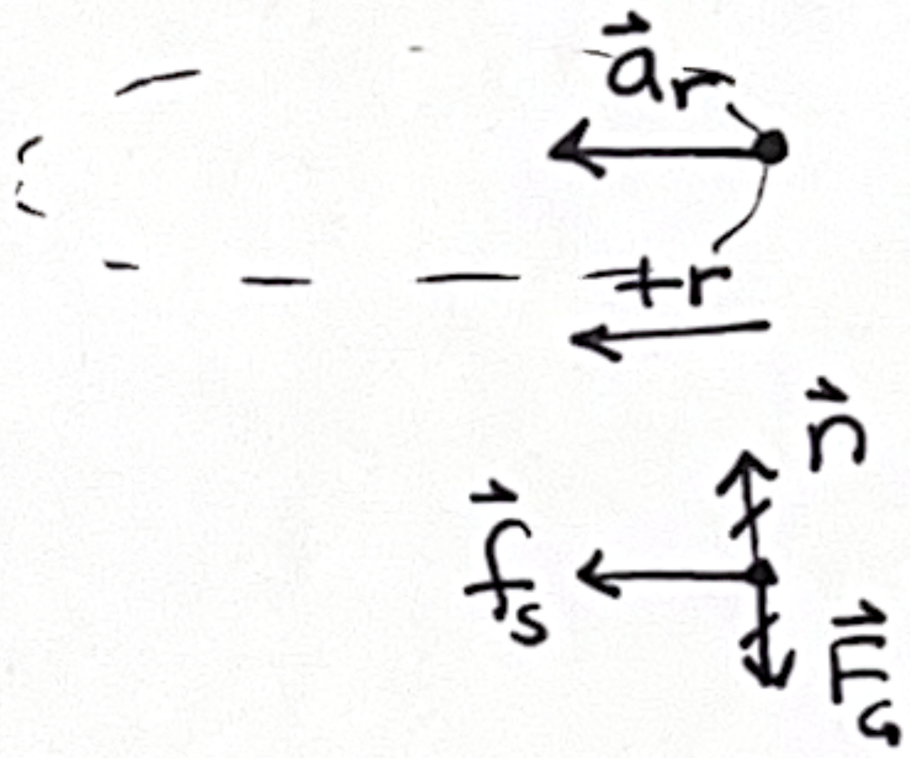
If the speed of the car was doubled, what would happen to the amount of static friction force needed to make the turn?

If the speed doubles, the static friction force needed will quadruple. This is because in my result, $f_s = M\left(\frac{V_o^2}{R}\right)$, the static friction force depends on the square of the speed.

We can also see it this way: if speed is $2v_o$, then $f_{s_{new}} = M\frac{(2v_o)^2}{R} = \frac{4Mv_o^2}{R}$, which is 4 times the original static friction force.

Problem 2: A car of mass M traveling on a flat road goes around a curve of radius R with the fastest possible speed v_{max}. The coefficient of friction between the road and tires is μ_s. Find an expression for the maximum speed of the car, v_{max}, in terms of given variables and fundamental constants.

Sketch and translate • Sketch the situation described. Label all relevant information. • Choose the system and a specific position to analyze. • Identify the unknown that you need to find and label it with a question mark.	
Simplify and diagram • Decide if the system can be modeled as a point-like object. — yes • Determine if the constant speed circular motion approach is appropriate. — yes • Indicate with an arrow the direction of the object's acceleration as it passes the chosen position. • Draw an FBD for the system at the instant it passes that position. • Beside the FBD, draw your +r-axis	
Represent mathematically • Convert the force diagram into the radial r-component form of Newton's second law. • For objects moving in a horizontal circle you may also need to apply a vertical y-component form of Newton's second law	$\Sigma F_r = ma_r$ $f_{s\,max} = M\left(\frac{V_{max}^2}{R}\right)$ $\mu_s n = \frac{M V_{max}^2}{R}$ $\Sigma F_y = ma_y$ $n - F_G = 0$ $n - Mg = 0$ $n = Mg$
Solve and evaluate • Solve the equations formulated in the previous step.	I need bothe equations because the y-direction tells me the normal force. Substituting the normal force: $\mu_s(\cancel{M}g) = \frac{\cancel{M} V_{max}^2}{R}$ $\mu_s g = \frac{V_{max}^2}{R}$

$$\boxed{\sqrt{\mu_s g R} = V_{max}}$$

How does increasing the mass of the car or radius of the curve affect the possible maximum speed?

- Since $V_{max} = \sqrt{\mu_s g R}$, and mass does not show up in this equation, increasing the mass does not affect V_{max}.
- But if the radius is increased, V_{max} also increases because $V_{max} = \sqrt{\mu_s g R}$.