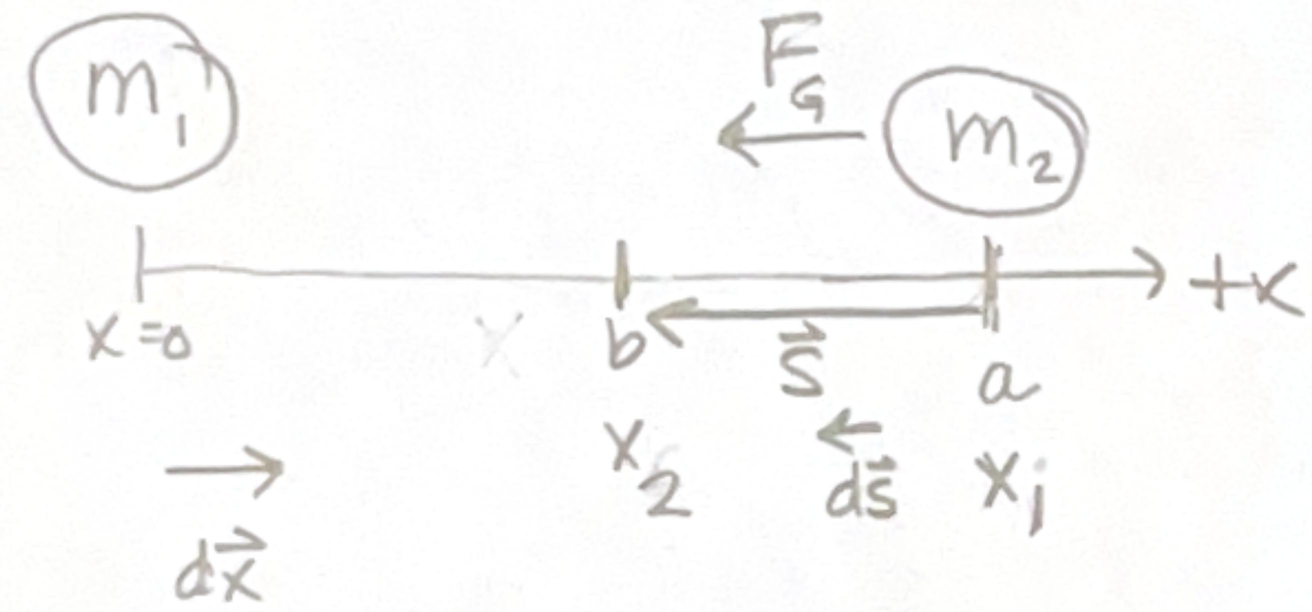
T5) Ch9 p.229 #53

a)



Find work done by F_G.
The force varies with distance, so I can't use $W = Fd\cos\theta$. I have to use $W = \int_a^b \vec{F}\cdot d\vec{s}$.

$$W = \int_a^b \vec{F}\cdot d\vec{s}$$

$$W = \int_a^b \left(\frac{Gm_1m_2}{x^2}\right)(ds)\cos 0$$

$$W = \int_{x_1}^{x_2} \frac{Gm_1m_2}{x^2}(-dx)$$

$$W = -Gm_1m_2 \int_{x_1}^{x_2} (x^{-2})\,dx$$

$$W = -Gm_1m_2\left(\frac{1}{-2+1}x^{-2+1}\right)\Big|_{x_1}^{x_2}$$

$$W = -Gm_1m_2\left(-\frac{1}{x}\right)\Big|_{x_1}^{x_2}$$

$$W = \frac{Gm_1m_2}{x}\Big|_{x_1}^{x_2}$$

$$\boxed{W = \frac{Gm_1m_2}{x_2} - \frac{Gm_1m_2}{x_1}}$$

Then find the U(x) function:

$$\Delta U = -W$$

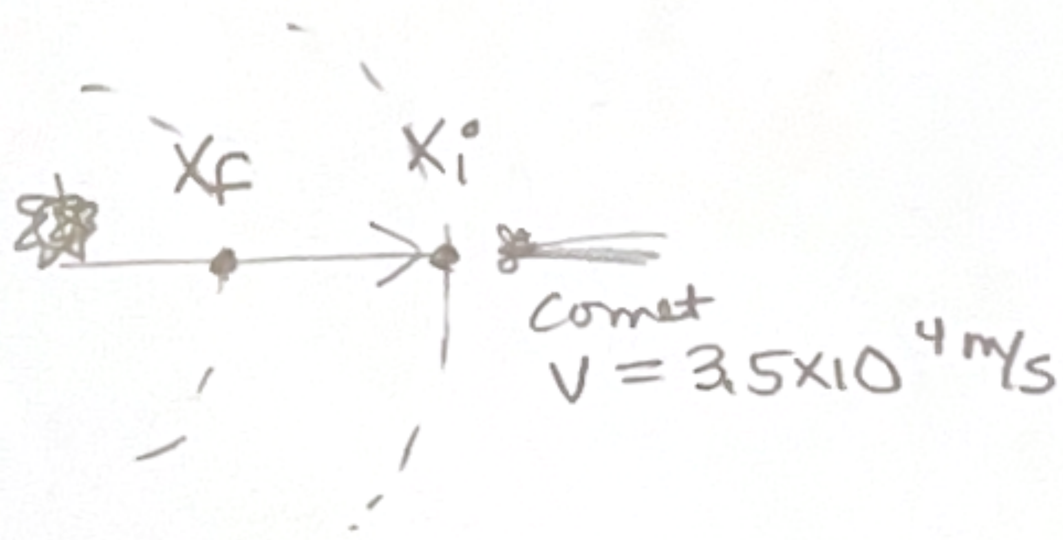
$$U(b) - U(a) = -\left(\frac{Gm_1m_2}{x_2} - \frac{Gm_1m_2}{x_1}\right)$$

$$U(x_2) - U(x_1) = -\frac{Gm_1m_2}{x_2} + \frac{Gm_1m_2}{x_1}$$

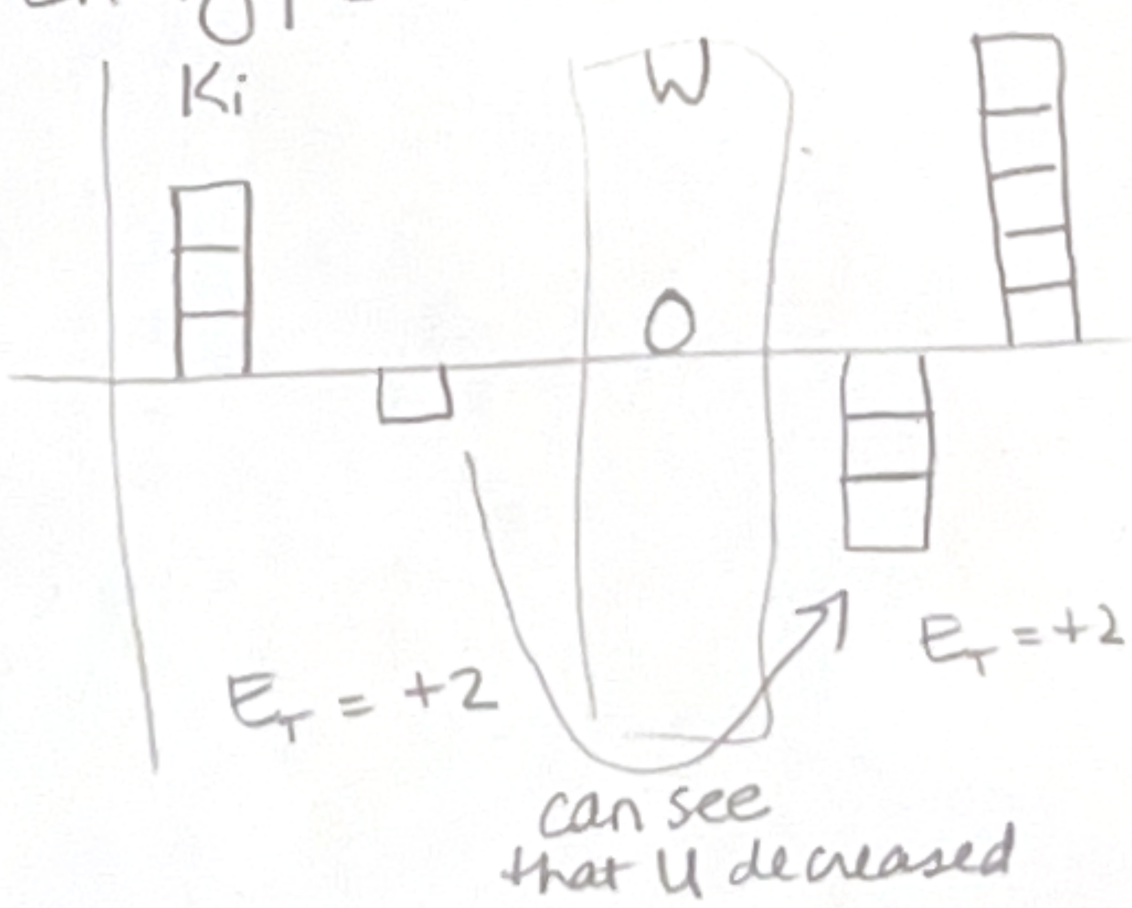
$$U(x_2) - U(x_1) = -\frac{Gm_1m_2}{x_2} - \left(-\frac{Gm_1m_2}{x_1}\right)$$

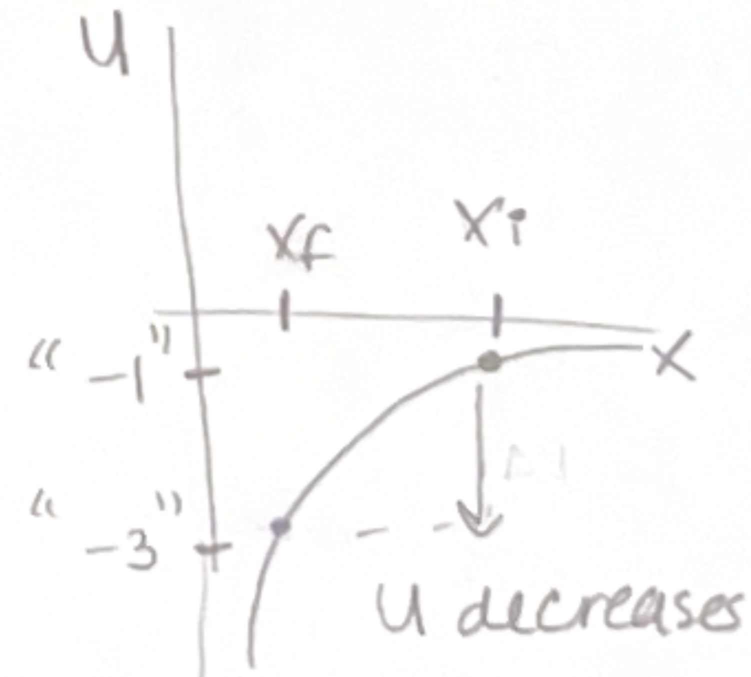
So $\boxed{U(x) = -\frac{Gm_1m_2}{x}}$

b)



Energy Bar Chart: no work done by ext. forces





radius of Mercury's orbit = 5.79×10^{10} m
radius of mars' orbit = 2.28×10^{11} m
V_i comet = 3.5×10^4 m/s
mass of sun = 1.99×10^{30} kg
mass of comet = 1.5×10^{13} kg

#53b, continued

Energy equation: $K_i + U_{Gi} = K_f + U_{Gf}$

$$\frac{1}{2}m_c V_{ci}^2 + -\frac{G m_s m_c}{r_{mars\ orbit}} = \frac{1}{2}m_c V_{cf}^2 + -\frac{G m_s m_c}{r_{Mercury\ orbit}}$$

all the m_c's cancel out

$$\frac{1}{2}V_{ci}^2 - \frac{G m_s}{r_{mars\ orbit}} = \frac{1}{2}V_{cf}^2 - \frac{G m_s}{r_{merc.\ orbit}}$$

multiply all by 2

$$V_{ci}^2 - \frac{2G m_s}{r_{mars\ orbit}} = V_{cf}^2 - \frac{2G m_s}{r_{merc.\ orbit}}$$

Simplify

$$V_{ci}^2 - \frac{2G m_s}{r_{mars}} + \frac{2G m_s}{r_{merc.}} = V_{cf}^2$$

$$V_{ci}^2 - 2G m_s\left(\frac{1}{r_{mars}} - \frac{1}{r_{merc.}}\right) = V_{cf}^2$$

put in values

$$(3.5\times10^4\,m/s)^2 - 2\left(6.67\times10^{-11}\,\frac{Nm^2}{kg^2}\right)(1.99\times10^{30})\left(\frac{1}{2.28\times10^{11}\,m} - \frac{1}{5.79\times10^{10}\,m}\right) = V_{cf}^2$$

calculate answer

$$\boxed{6.8\times10^4\,m/s} = V_{cf}$$

<u>Note</u>: I believe the book has the wrong answer in the back.